Projection Based Model Order Reduction for Multiphysical Problems

Modes, Load Vectors, Couplings

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Modes
Fourier

Geometry: Circle edge, periodic interval $\varphi \in [0, 2\pi]$

- Basis: $[\sqrt{1/2\pi}, \sqrt{1/\pi} \cdot \sin \varphi, \sqrt{1/\pi} \cdot \cos \varphi, \sqrt{1/\pi} \cdot \sin 2 \varphi, \sqrt{1/\pi} \cdot \cos 2 \varphi ...]$

- Example: Force density in air gap of induction machine
Legendre

Geometry: Finite straight line \( x \in [-1, 1] \)

- Basis: Legendre Polynomials

\[
\begin{align*}
P_0(x) &= 1 \\
P_1(x) &= x \\
P_2(x) &= \frac{1}{2}(3x^2 - 1) \\
P_3(x) &= \frac{1}{2}(5x^3 - 3x) \\
P_4(x) &= \frac{1}{8}(35x^4 - 30x^2 + 3) \\
P_5(x) &= \frac{1}{8}(63x^5 - 70x^3 + 15x) \\
P_6(x) &= \frac{1}{16}(231x^6 - 315x^4 + 105x^2 - 5)
\end{align*}
\]

- Example:
  Normal deformation of sliding rail of machine tool
Fourier + Legendre

Geometry: Cylindric Surface $\varphi, z \in [0, 2\pi] \times [-l/2, l/2]$

- Basis: $[F_i(\varphi) \cdot P_j(z)]$
- Example: Force density in air gap of claw pole machine
Radial Polynomia + Fourier + Legendre

Geometry: Hollow Cylinder \( r, \varphi, z \in [R_{in}, R_{out}] \times [0, 2\pi] \times [-l/2, l/2] \)

- Basis: \([R_i(r) \cdot F_j(\varphi) \cdot P_k(z)]\)
- Example: For rotating disk we take \( F_j(\varphi) = 1 \), combination of radial and axial polynomia projects Joule heat as axisymmetric onto hollow cylinder.
Zernike

Geometry: Circle

• Basis: Zernike Polynomials

\[ Z^m_n(\rho, \varphi) = R^m_n(\rho) \cos(m \varphi) \]

\[ Z^{-m}_n(\rho, \varphi) = R^m_n(\rho) \sin(m \varphi), \]

\[ R^m_n(\rho) = \sum_{k=0}^{\frac{n-m}{2}} \frac{(-1)^k (n-k)!}{k! \left( \frac{n+m}{2} - k \right)! \left( \frac{n-m}{2} - k \right)!} \rho^{n-2k} \]

• Example: Optical property of lens during transient thermal load
Laplace’s Spherical Harmonics

Geometry: Sphere

• Basis:

\[ Y_{\ell}^{m}(\theta, \varphi) = Ne^{im\varphi} P_{\ell}^{m}(\cos \theta) \]

\[ P_{\ell}^{m} : [-1, 1] \rightarrow \mathbb{R} \] is an associated Legendre polynomial

• Example: Expansion of acoustic irradiation

Mulit-Pole expansion
Thermal Mode Example

Modes from GSO of selected Transient Results
Thermal Mode Example

Modes from MOS of Transient Results
Load Vectors and State Space Reduction
Conservative System Model

How do conservative terminals behave?

\[ u_1 \cdot A_1 + u_2 \cdot A_2 + u_3 \cdot A_3 = u_m \cdot A_m \]

\[ w_1 \cdot u_1 + w_2 \cdot u_2 + w_3 \cdot u_3 = u_m \]

\[ F_m = A_m \cdot p \]

\[ F_1 = A_1 \cdot p = w_1 \cdot F_m \]

\[ F_2 = A_2 \cdot p = w_2 \cdot F_m \]

\[ F_3 = A_3 \cdot p = w_3 \cdot F_m \]

\[ W^T \cdot u = u_m \]

\[ F = W \cdot F_m \]

Input- and output matrices are mutually transposed for conservative systems.
Reduction with Load Vectors

Modal Reduction

In postprocessing of modal analysis:
Append load vectors to files of modes
Create state space model

Theory

\[ C_R \cdot \hat{\theta} + K_R \cdot \hat{\theta} = V^T Q \]
\[ \theta = V \cdot \hat{\theta} \]

\[ E \cdot \dot{x} + A \cdot x = B \cdot u \]
\[ y = C \cdot x \]

With the conservative case: \( C = B^T \)

What is needed?
• System matrices
• Vectors for reduction
• Load vectors

Krylov Reduction

With the conservative case: \( C = B^T \)

What is needed?
• System matrices
• Vectors for reduction
• Load vectors
SPMWRITE to Create Housing ROM

- Apply excitation and radiation force patterns as additional load vectors to modal file .mode
- Export state space matrices using SPMWRITE, transfer to SML
Load Vectors, Transfer Matrices to SML, Convert to Causal

- Load vectors are applied as spatial pressure distributions
- Allow definition of input loads (force waves onto tooth faces) and output loads (surface modes)
- SPMWR creates State Space Model, cut last columns from input and first rows from output
- New in Ansys 2024: SML file is directly written: `keyw,beta,1 $ spmwrite`

\[
\begin{align*}
\mathbf{x}' &= A\mathbf{x} + B\mathbf{u} \\
\mathbf{y} &= C\mathbf{x}'
\end{align*}
\]
Transient NVH Analysis

- Input frequency ramped to 900 Hz Speed ramped to 18000 RPM
- PWM frequency 6000 Hz
- Id and Iq are functions of ECE.pos and currents
- 3DTAB finds force wave coefficients
- Causal ROM transfers to surface waves
- Sum of surface velocities times impedance gives sound pressure
Live Example:

PCB Thermal MOR with Load Vectors
Simple Induction Heating Example

Field and Reduced Simulation
Simple 2D Induction Heating Example

Moving Inductor

Harmonic / Transient EM Solution
Transfer Heat Generation Rate to THER
Solve Transient THER Step with Restart
Transfer Temperature to EM

Flux Lines and Current Density

Transient Temperature Distribution
Field Coupling

Electromagnetic Analysis

Heat Generation Distribution

Temperature Distribution

Transient Thermal Analysis

- **Static** interaction: from actual temperature distribution the actual heat generation is produced
- **Nonlinear**: BH-curve, temperature dependent

- **Transient** behaviour: last time step is start for next
- **Linear**: PDE system with constant coefficients

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System Coupling

- **Static** interaction: from actual temperature coefficients, inductor position and current, the heat generation coefficients are found.
- **Nonlinear**: outputs are found from response surface calculation of inputs.
- **optiSLang** creates Metamodel of Optimal Prognosis.

- **Transient** behaviour: state space model.
- **Linear**: matrices A, B, C describe equation of motion.
- **MORiA** creates the ROM for TwinBuilder based on thermal system matrices.

Electromagnetic MOP

Heat Generation Coefficients

Temperature Coefficients

Thermal ROM
Temperature Distribution Projected onto Basis

TEMP-Coefficients vs. Time

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System Simulation in Twin Builder

- Task:
- Compare TEMP and HEAT coefficients to those generated by coupled field simulation
Reduced TEMP and HEAT Comparison

TEMP Coefficients

HEAT Coefficients
Summary

• Modes for all physical domains

• Modes for equation of motion

• Modes for coupling

• Modes as load vectors for ROM generation

• Modes as DOF in system simulation