Projection Based Model Order Reduction for Multiphysical Problems

Modes, Load Vectors, Couplings

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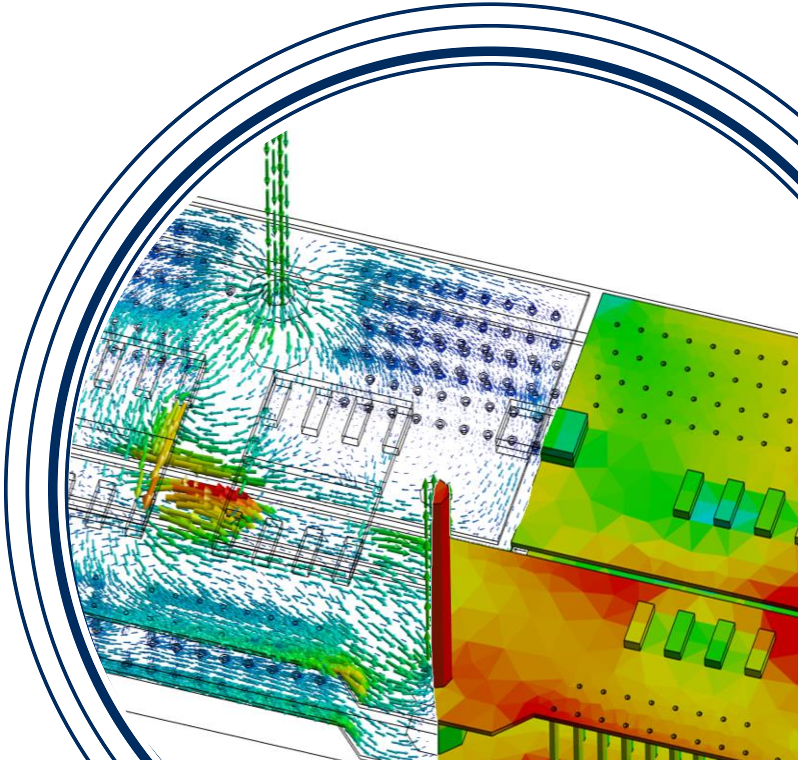
CADFEM Germany GmbH



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  - Thermal Examples: GSO, MOS
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  - Conservative Terminals
  - Load Vector Terminals
- Couplings
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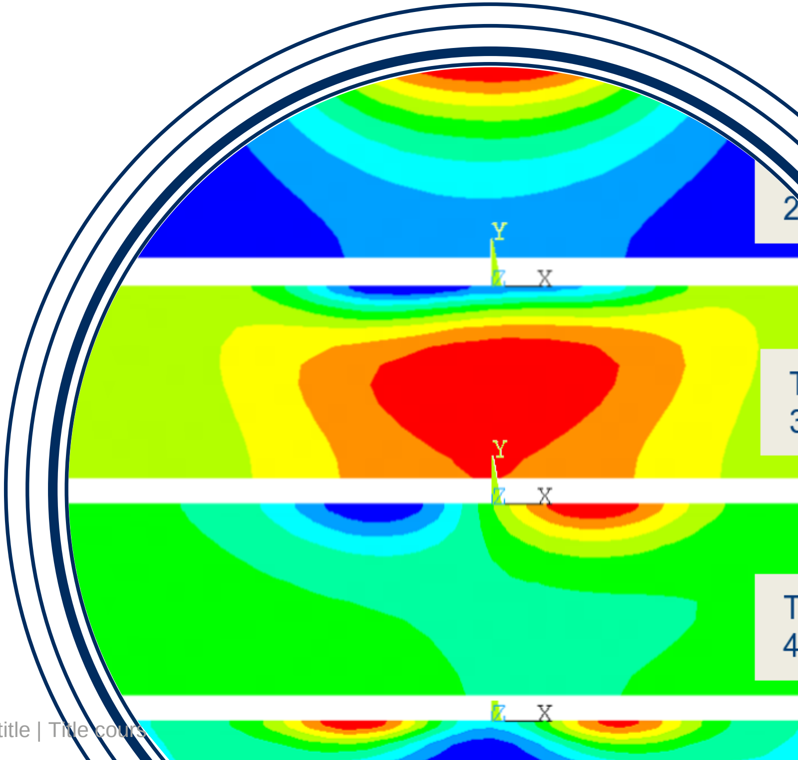
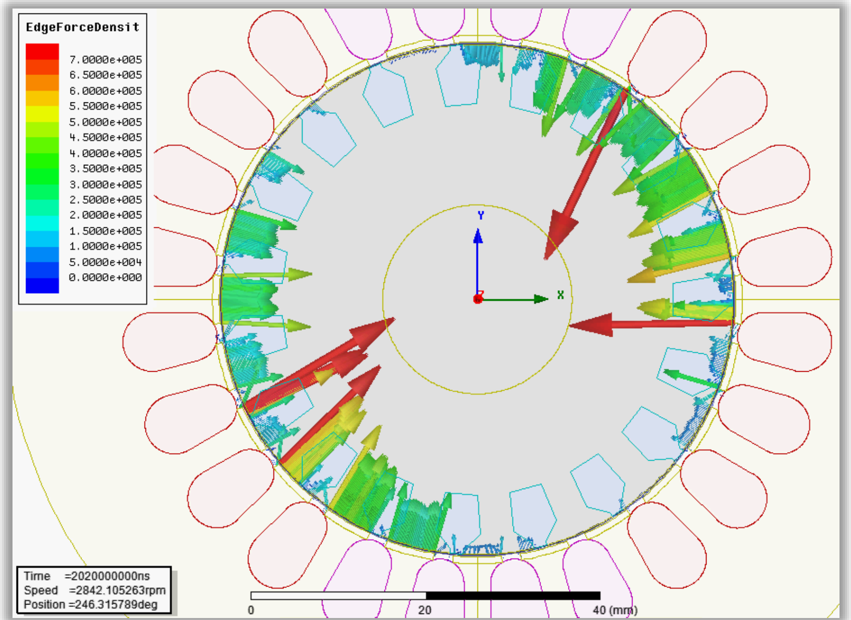
Modes

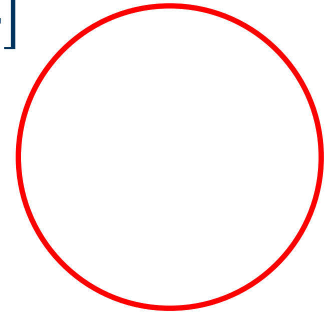


Fourier

Geometry: Circle edge, periodic interval $\varphi \in [0, 2\pi]$

- Basis: $\left[\sqrt{1/_{2\pi}}, \sqrt{1/_{\pi}} \cdot \sin\varphi, \sqrt{1/_{\pi}} \cdot \cos\varphi, \sqrt{1/_{\pi}} \cdot \sin 2\varphi, \sqrt{1/_{\pi}} \cdot \cos 2\varphi \ldots\right]$
- Example: Force density in air gap of induction machine

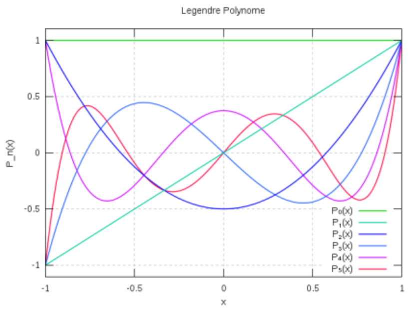




Legendre

Geometry: Finite straight line $\mathbf{x} \in [-1, 1]$

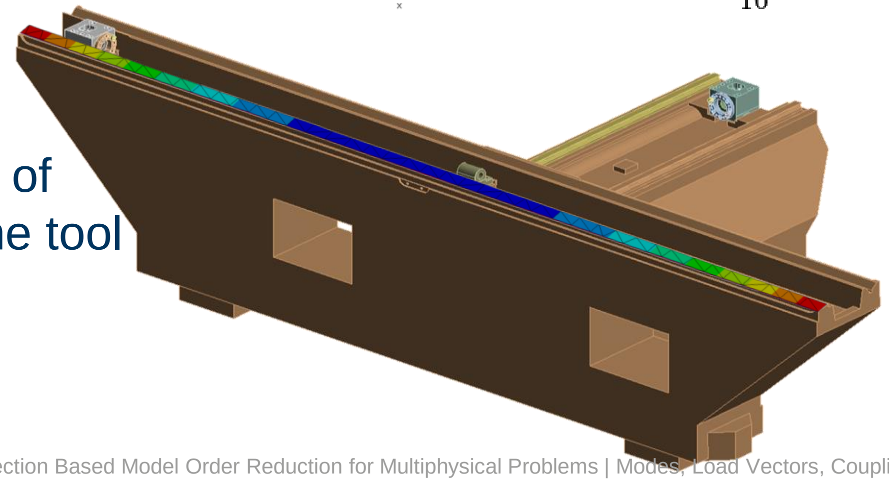
- Basis: Legendre Polynomia



$$P_0(x) = 1$$
$$P_1(x) = x$$
$$P_2(x) = \frac{1}{2}(3x^2 - 1)$$
$$P_3(x) = \frac{1}{2}(5x^3 - 3x)$$
$$P_4(x) = \frac{1}{8}(35x^4 - 30x^2 + 3)$$
$$P_5(x) = \frac{1}{8}(63x^5 - 70x^3 + 15x)$$
$$P_6(x) = \frac{1}{16}(231x^6 - 315x^4 + 105x^2 - 5)$$

- Example:

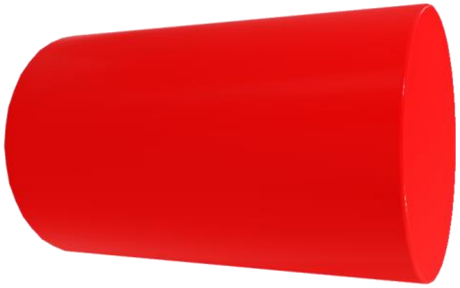
  Normal deformation of sliding rail of machine tool

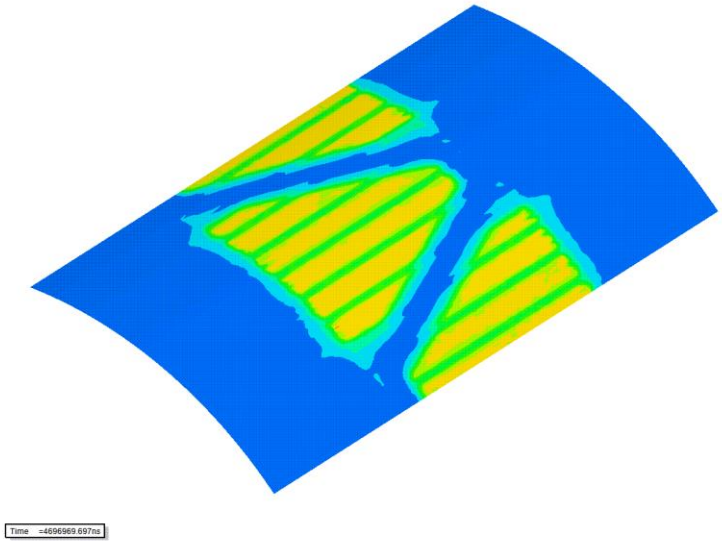


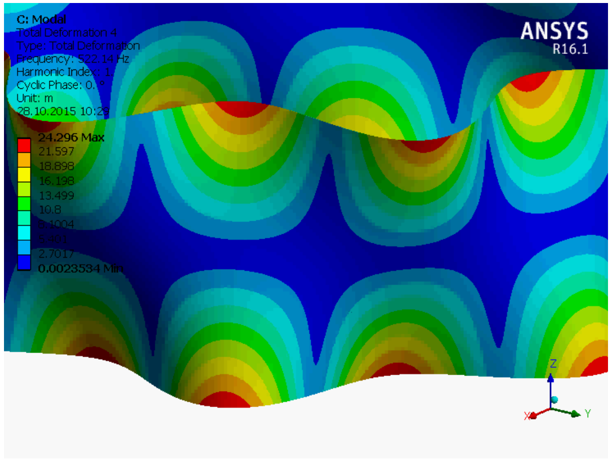
Fourier + Legendre

Geometry: Cylindric Surface $\varphi, z \in [0, 2\pi] \times [-l/2, l/2]$

- Basis: $[F_i(\varphi) \cdot P_j(z)]$
- Example: Force density in air gap of claw pole machine



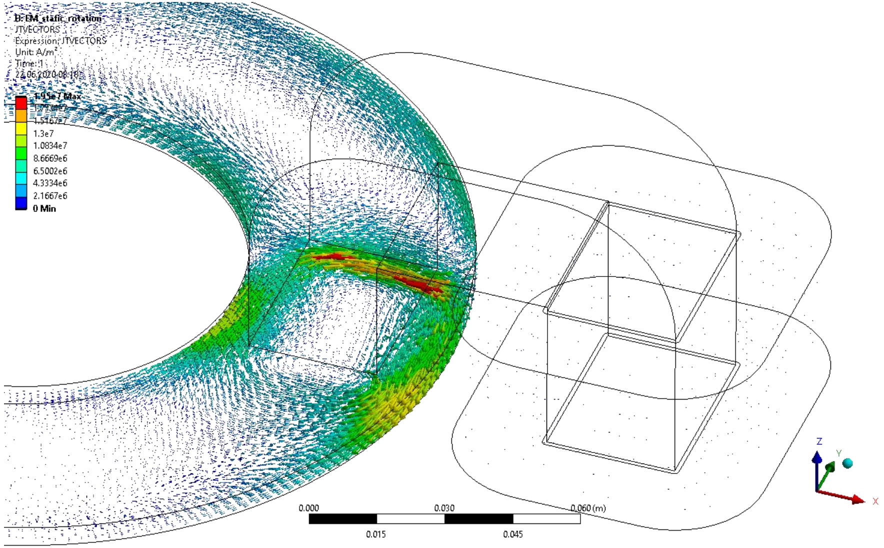


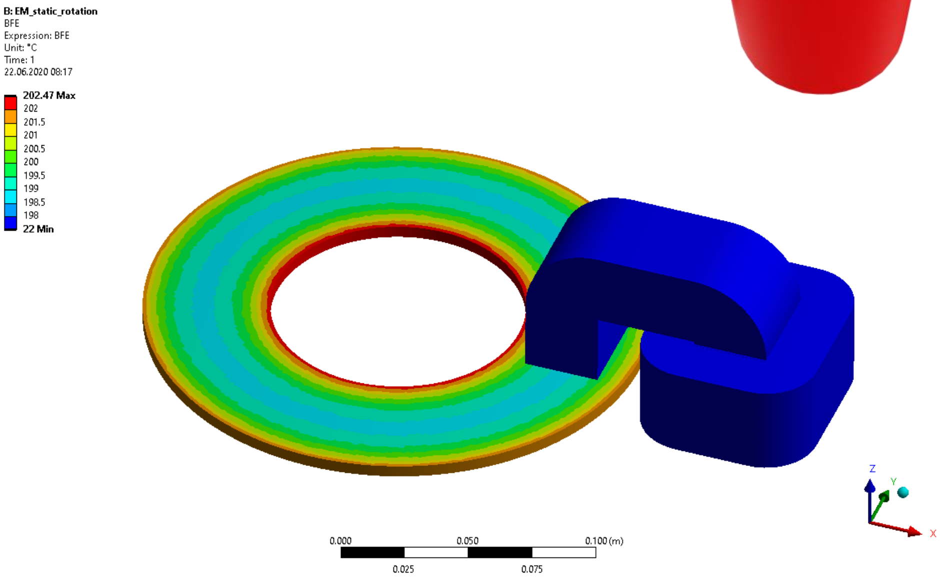


Radial Polynomia + Fourier + Legendre

Geometry: Hollow Cylinder $\boldsymbol{r}, \boldsymbol{\varphi}, \boldsymbol{z} \in [\boldsymbol{R_{in}}, \boldsymbol{R_{out}}] \times [\boldsymbol{0}, \boldsymbol{2\pi}] \times [-\boldsymbol{l}/\boldsymbol{2}, \boldsymbol{l}/\boldsymbol{2}]$

- Basis: $\left[R_i(r) \cdot F_j(\varphi) \cdot P_k(z)\right]$
- Example: For rotating disk we take $F_j(\varphi) = 1$, combination of radial and axial polynomia projects Joule heat as axisymmetric onto hollow cylinder.





Zernike

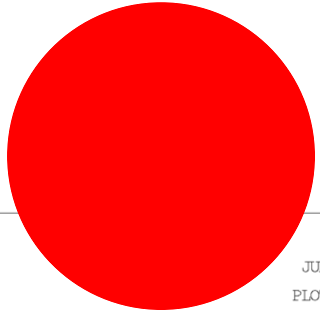
Geometry: Circle

- Basis: Zernike Polynomia

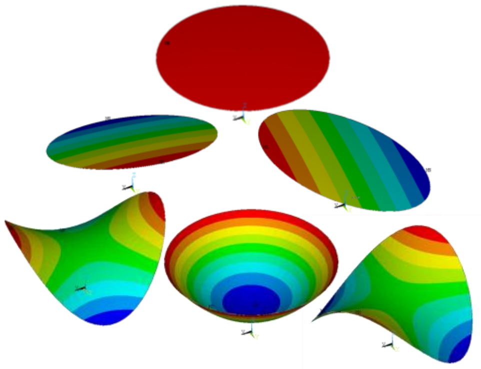
$$Z_n^m(\rho,\varphi) = R_n^m(\rho)\,\cos(m\,\varphi)$$

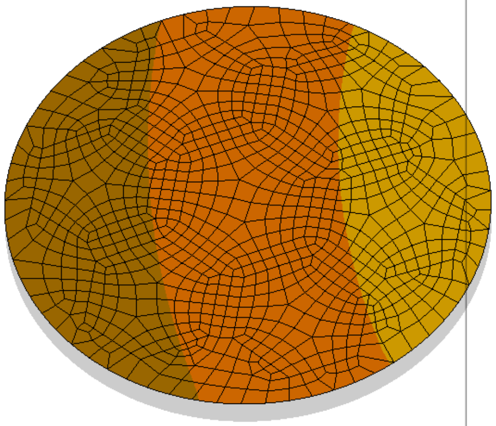
$$Z_n^{-m}(\rho,\varphi) = R_n^m(\rho)\,\sin(m\,\varphi),$$

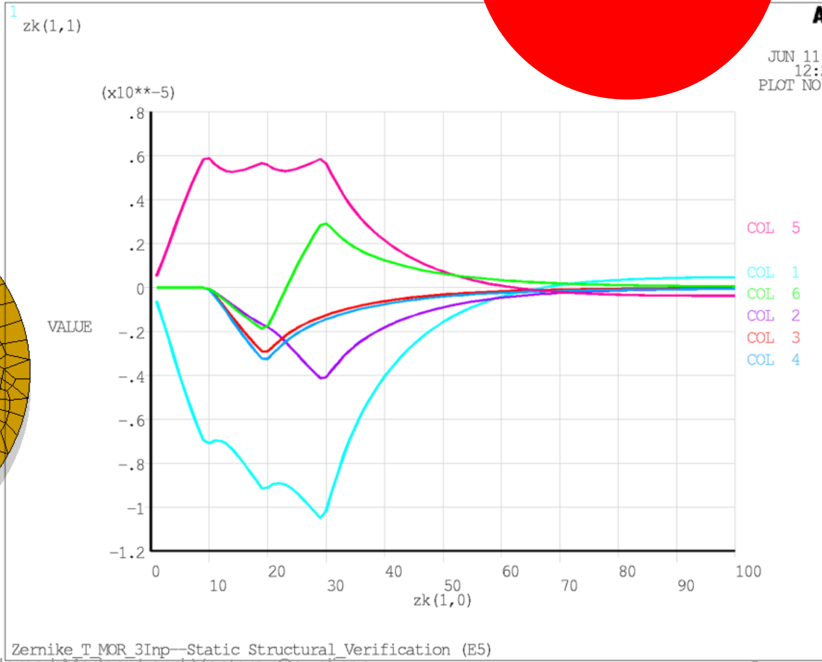
$$R_n^m(\rho) = \sum_{k=0}^{\frac{n-m}{2}} \frac{(-1)^k\,(n-k)!}{k!\,\left(\frac{n+m}{2}-k\right)!\,\left(\frac{n-m}{2}-k\right)!}\,\rho^{n-2k}$$



- Example: Optical property of lens during transient thermal load







Laplace‘s Spherical Harmonics

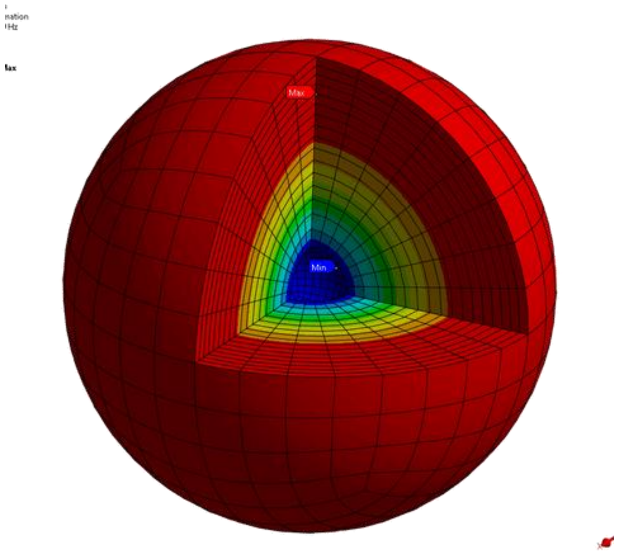
Geometry: Sphere

- Basis: $Y_\ell^m(\theta,\varphi) = N e^{im\varphi} P_\ell^m(\cos\theta)$

  $P_\ell^m : [-1,1] \to \mathbb{R}$ is an associated Legendre polynomial

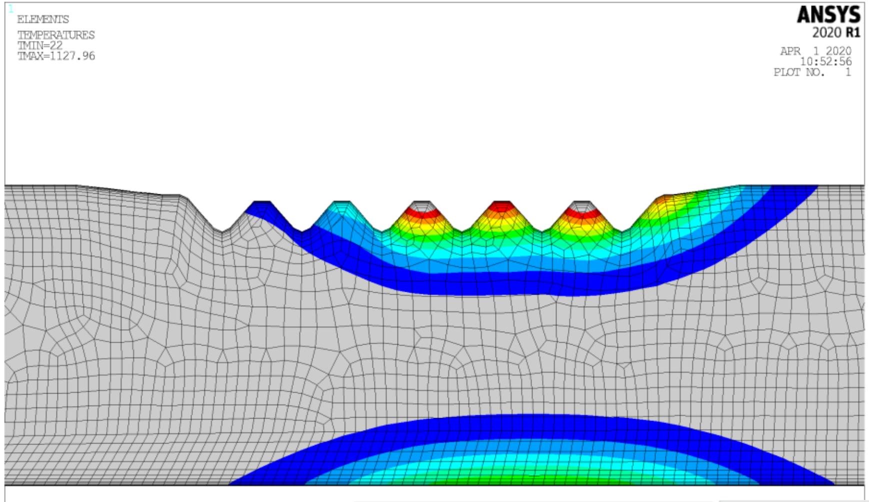
- Example: Expansion of acoustic irradiaton

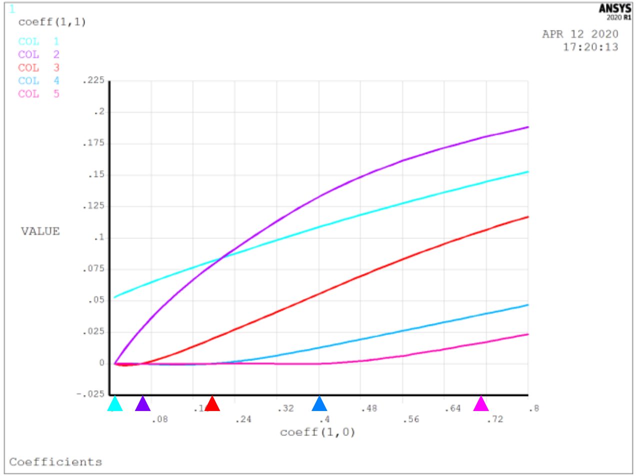
  Mulit-Pole expansion

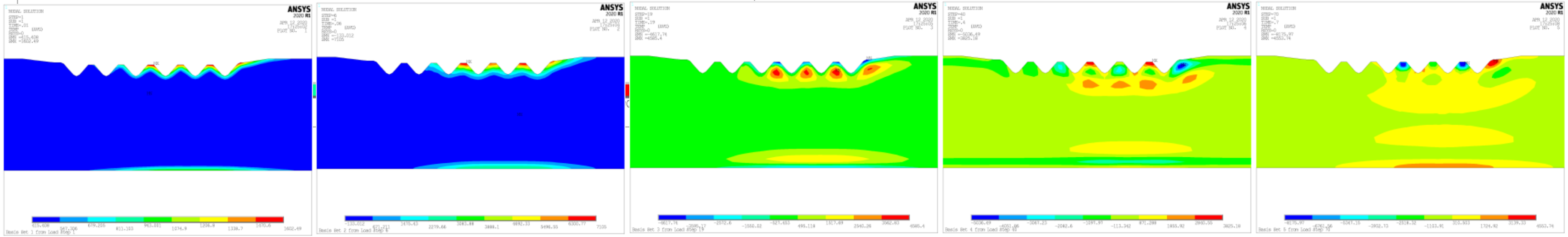


Thermal Mode Example

Modes from GSO of selected Transient Results

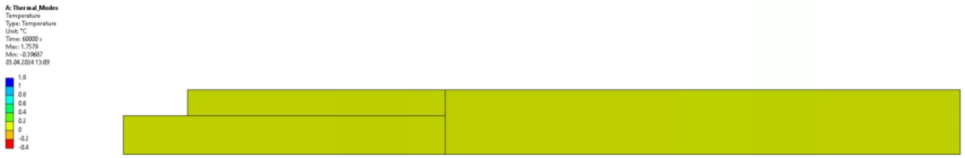


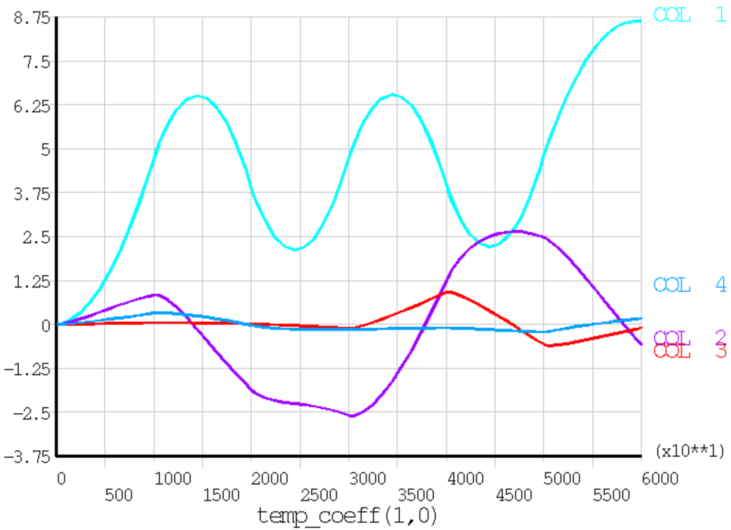


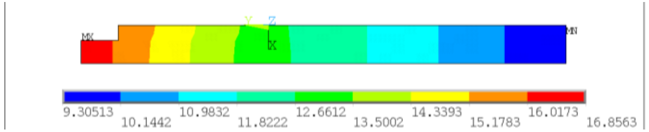


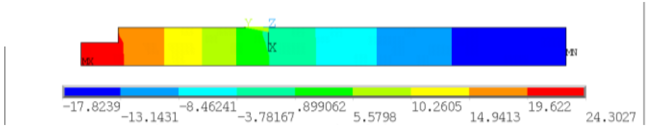
Thermal Mode Example

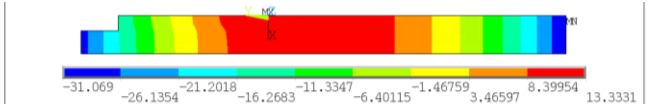
Modes from MOS of Transient Results

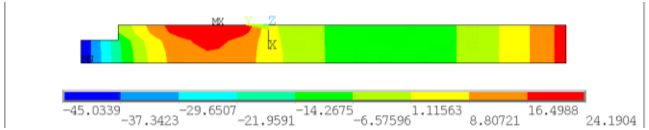




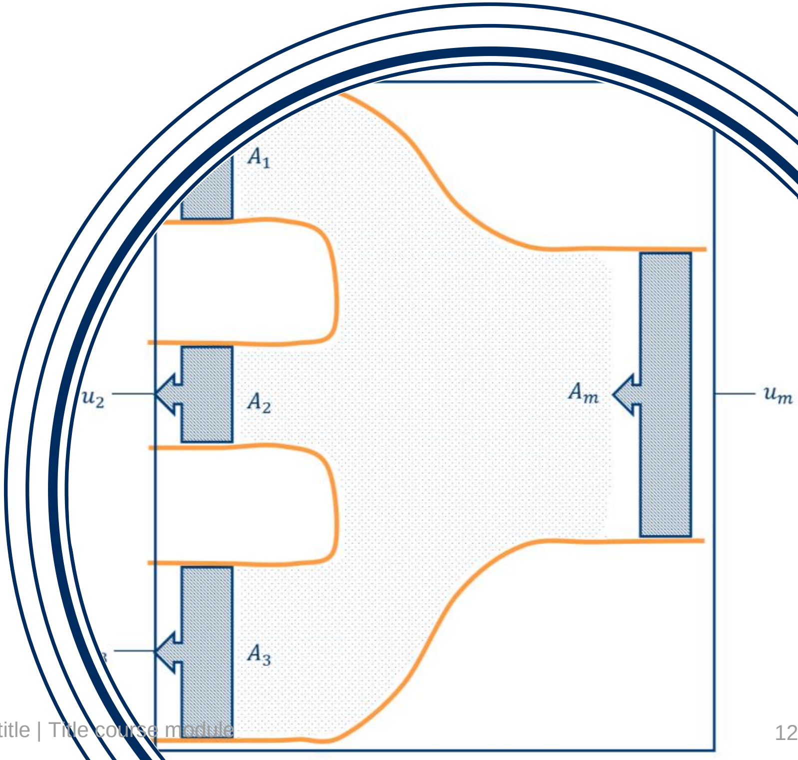






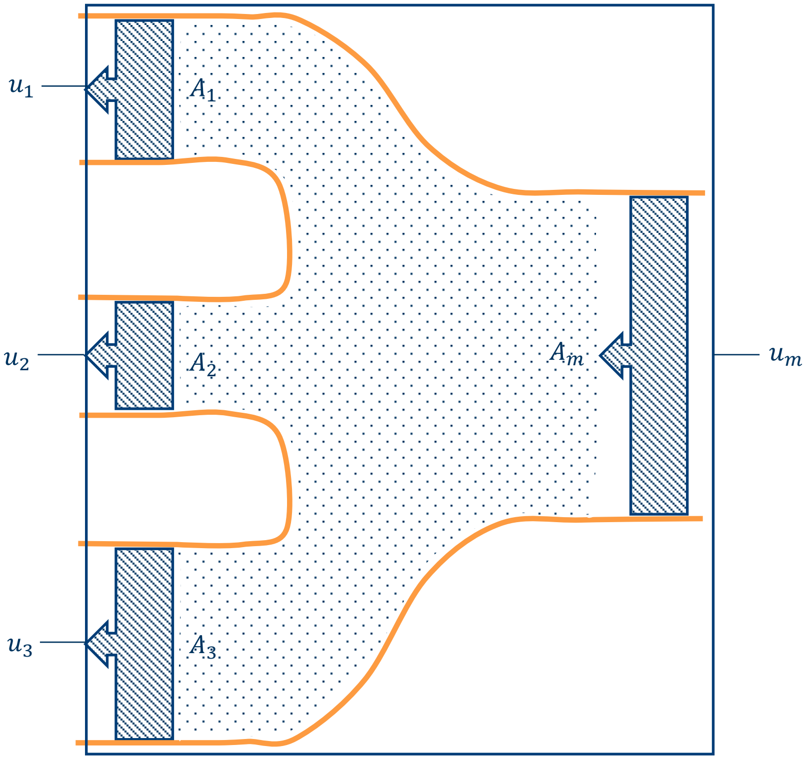


Load Vectors and State Space Reduction



Conservative System Model

How do conservative terminals behave?



$$u_1 \cdot A_1 + u_2 \cdot A_2 + u_3 \cdot A_3 = u_m \cdot A_m$$
$$w_1 \cdot u_1 + w_2 \cdot u_2 + w_3 \cdot u_3 = u_m$$

$$F_m = A_m \cdot p$$
$$F_1 = A_1 \cdot p = w_1 \cdot F_m$$
$$F_2 = A_2 \cdot p = w_2 \cdot F_m$$
$$F_3 = A_3 \cdot p = w_3 \cdot F_m$$

$$\mathbf{W}^{\mathrm{T}} \cdot \mathbf{u} = u_m$$
$$\mathbf{F} = \mathbf{W} \cdot F_m$$

Input- and output matrices are mutually transposed for conservative systems

Reduction with Load Vectors

Modal Reduction

In postprocessing of modal analysis:

Append load vectors to files of modes

Create state space model

```
/solu
modcont,on
mxpand,12,,,yes

*do,i,1,3
  ...
  sffu,pres,nfpress(1)
  sf,force_face,pres,0
  solve
  sfdel,all,all
*enddo

...

*do,i,1,7
  ...
  sffu,pres,nrpress(1)
  sf,rad_face,pres,0
  solve
  sfdel,all,all
*enddo

/post1
spmwrite,,,,,,,,,0
```

Theory

$$C_R \cdot \dot{\hat{\boldsymbol{\theta}}} + K_R \cdot \hat{\boldsymbol{\theta}} = \mathbf{V}^T \mathbf{Q}$$
$$\boldsymbol{\theta} = \mathbf{V} \cdot \hat{\boldsymbol{\theta}}$$

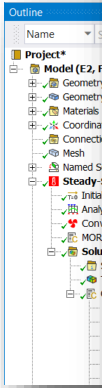
$$\mathbf{E} \cdot \dot{\mathbf{x}} + \mathbf{A} \cdot \mathbf{x} = \mathbf{B} \cdot \mathbf{u}$$
$$\mathbf{y} = \mathbf{C} \cdot \mathbf{x}$$

With the conservative case: $\mathbf{C} = \mathbf{B}^T$

What is needed?

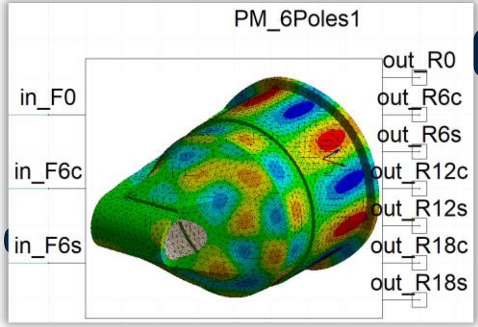
- System matrices
- Vectors for reduction
- Load vectors

Krylov Reduction

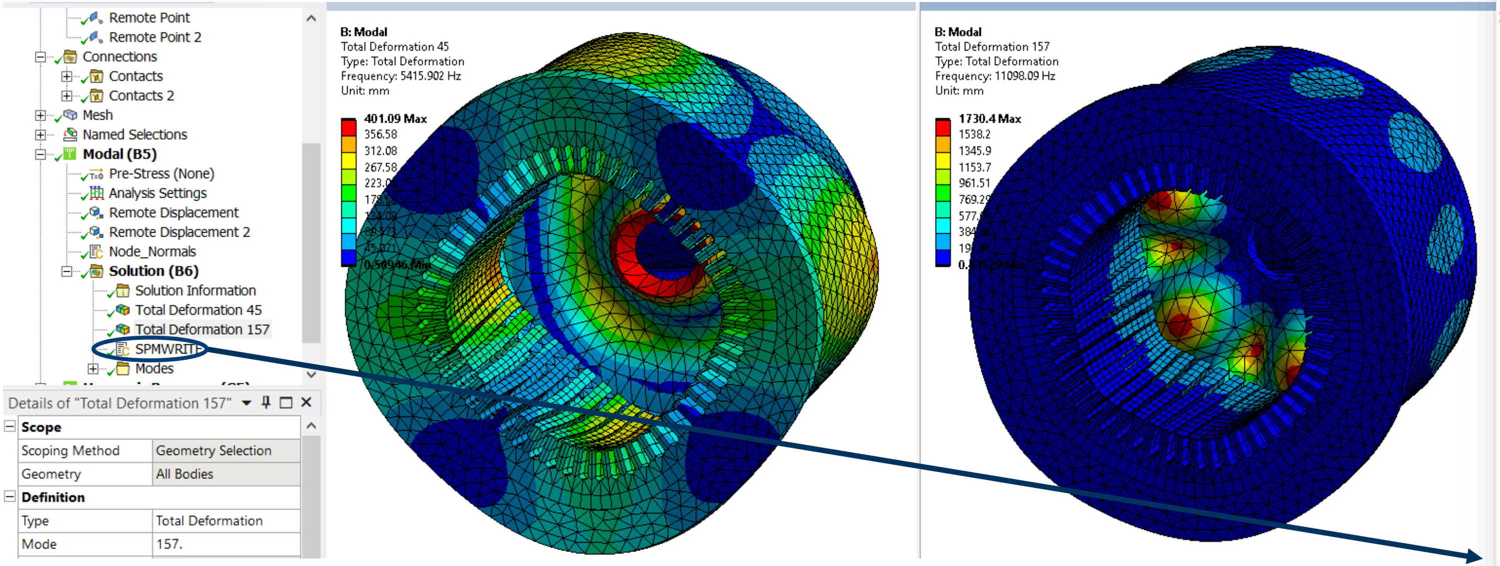



```
/com,Create System Matrices
harfrq,1/2/3.141592653589793
wrfull,1
solve
...

/com,Create Load Vectors
harfrq,0
wrfull,1
bfe,FET_body,hgen,,1/5.64056e-9   ! 1 Watt
solve
...

harfrq,0
wrfull,1
  *do,ie,1,emax
    bfe,ie,hgen,,evol(ie)*(ejvect(ie)**2      &
    +ejvect(ie+emax)**2+ejvect(ie+2*emax)**2)
  *enddo
solve
...

/com,Create Reduced Matrices
/sys,runmor.cmd
...

/com,Write System Model
! In Python:
sys = MakeFirstOrderSystem(
        MatrixE(sys),
        MatrixA(sys),
        MatrixB(sys),
        MatrixB(sys).transpose())
```

SPMWRITE to Create Housing ROM



- Apply excitation and radiation force patterns as additional load vectors to modal file .mode
- Export state space matrices using SPMWRITE, transfer to SML




```
/solu
modcont,on
mxpand,12,,,yes

*do,i,1,3
  ...
  sffu,pres,nfpress(1)
  sf,force_face,pres,0
  solve
  sfdel,all,all
*enddo

...

*do,i,1,7
  ...
  sffu,pres,nrpress(1)
  sf,rad_face,pres,0
  solve
  sfdel,all,all
*enddo

/post1
spmwrite,,,,,,,,,0
```

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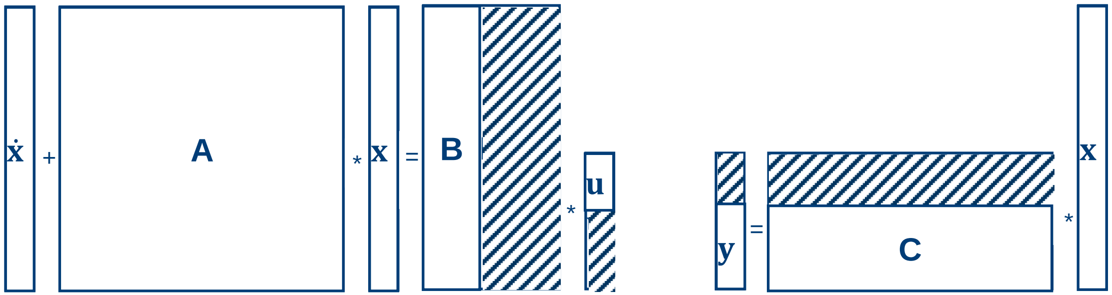
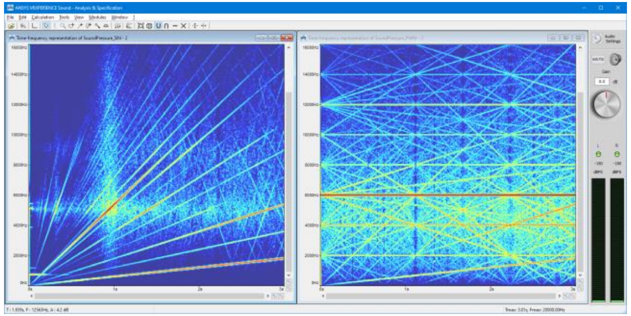
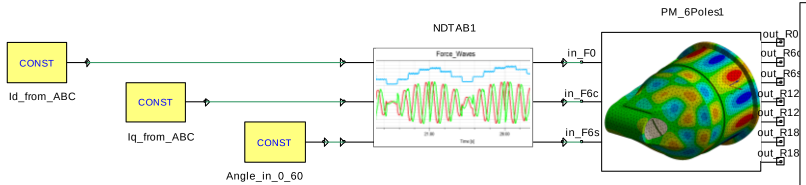
Load Vectors, Transfer Matrices to SML, Convert to Causal

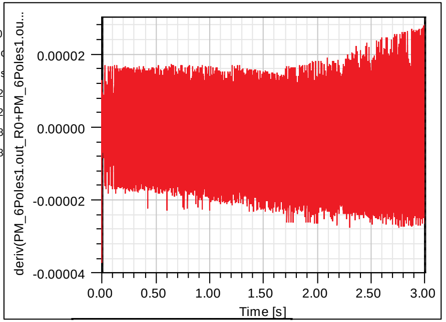
- Load vectors are applied as spatial pressure distributions
- Allow definition of input loads (force waves onto tooth faces) and output loads (surface modes)
- SPMWR creates State Space Model, cut last columns from input and first rows from output
- New in Ansys 2024: SML file is directly written: `keyw,beta,1 $ spmwrite`

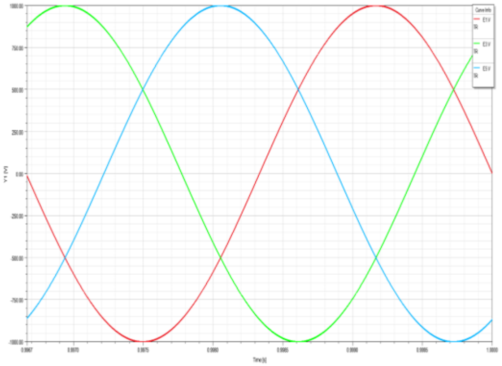


Transient NVH Analysis

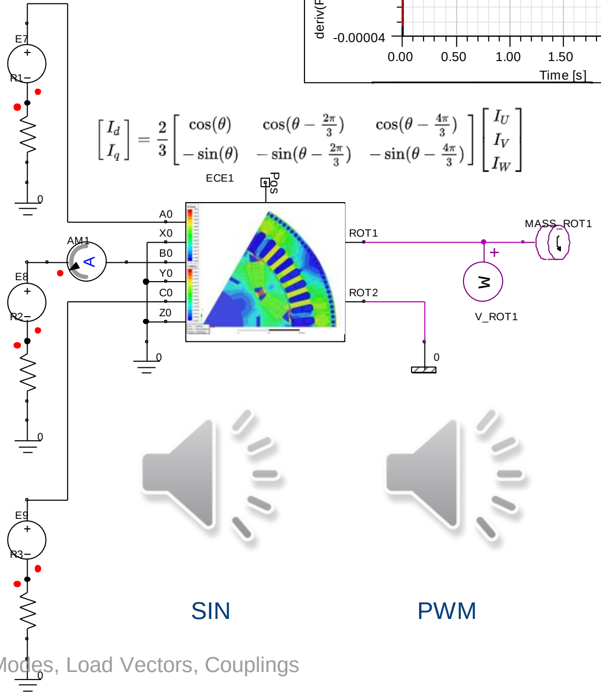








$$\begin{bmatrix} I_d \\ I_q \end{bmatrix} = \frac{2}{3}\begin{bmatrix} \cos(\theta) & \cos(\theta-\frac{2\pi}{3}) & \cos(\theta-\frac{4\pi}{3}) \\ -\sin(\theta) & -\sin(\theta-\frac{2\pi}{3}) & -\sin(\theta-\frac{4\pi}{3}) \end{bmatrix}\begin{bmatrix} I_U \\ I_V \\ I_W \end{bmatrix}$$



SIN PWM

- Input frequency ramped to 900 Hz Speed ramped to 18000 RPM
- PWM frequency 6000 Hz

- Id and Iq are functions of
- ECE.pos and currents

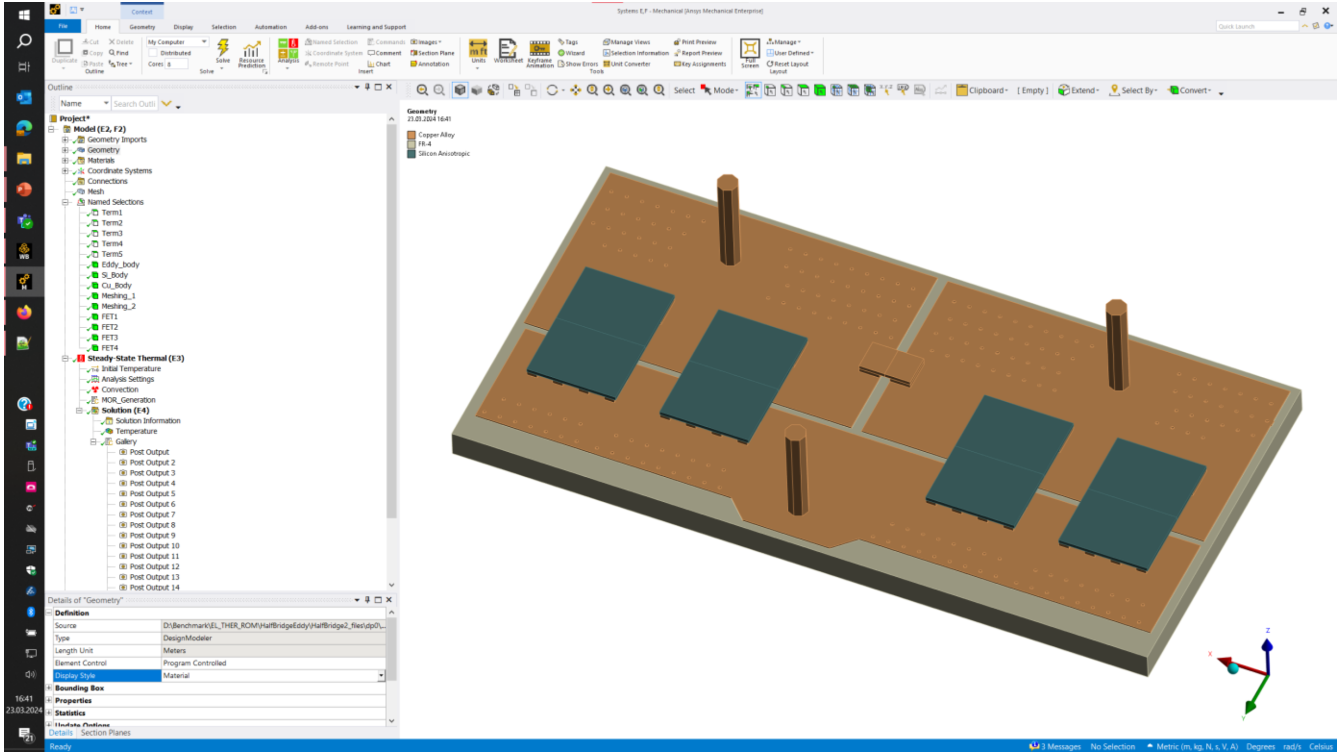
- 3DTAB finds force wave coefficients

- Causal ROM transfers to surface waves

- Sum of surface velocities times impedance gives sound pressure

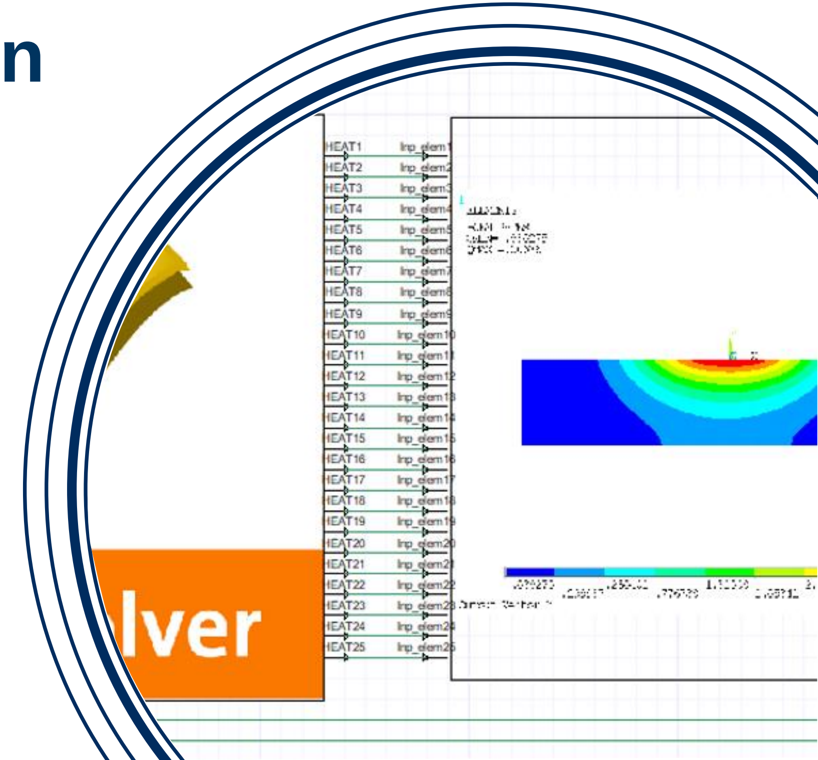
Live Example:

PCB Thermal MOR with Load Vectors



Simple Induction Heating Example

Field and Reduced Simulation



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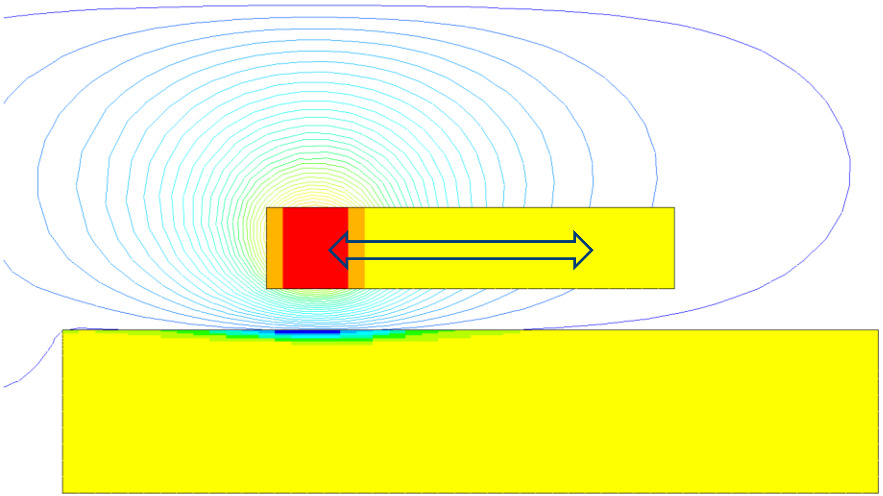
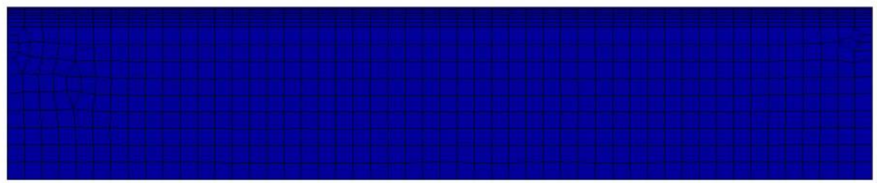
Simple 2D Induction Heating Example

Moving Inductor



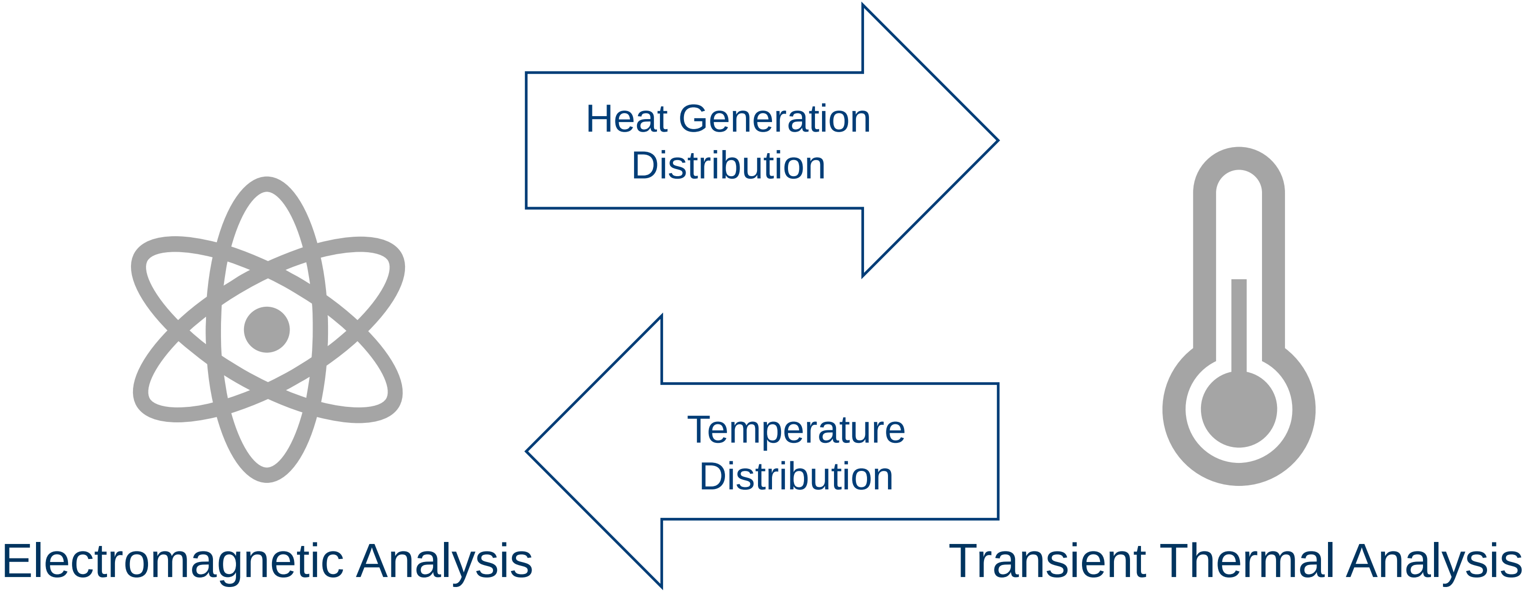
Flux Lines and Current Density

Harmonic / Transient EM Solution
Transfer Heat Generation Rate to THER
Solve Transient THER Step with Restart
Transfer Temperature to EM



Transient Temperature Distribution

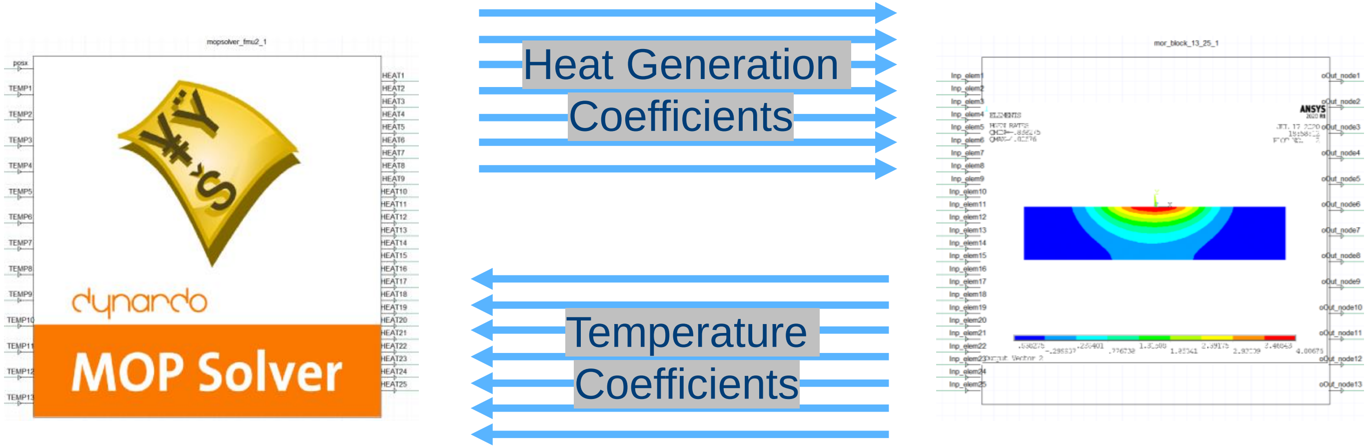
Field Coupling



- **Static** interaction: from actual temperature distribution the actual heat generation is produced
- **Nonlinear**: BH-curve, temperature dependent

- **Transient** behaviour: last time step is start for next
- **Linear**: PDE sytem with constant coefficients

System Coupling



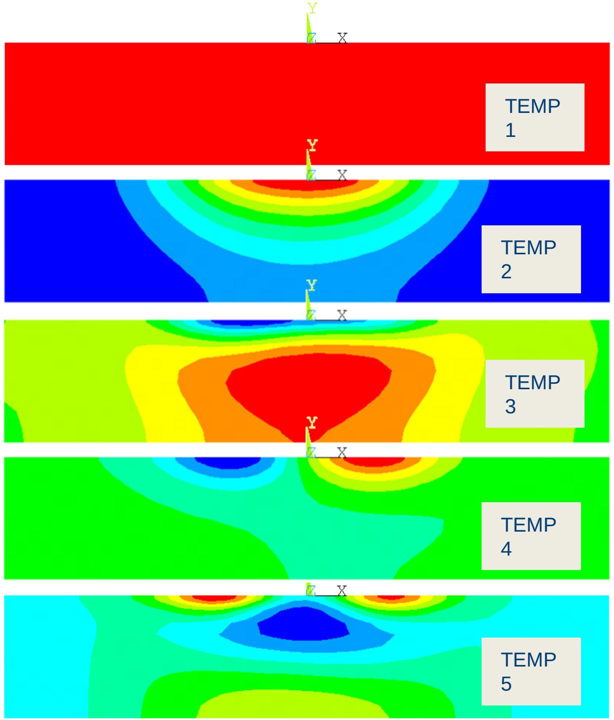
Electromagnetic MOP

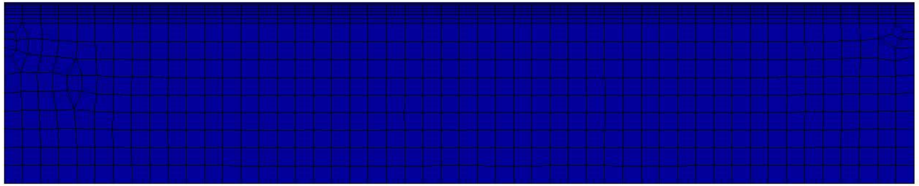
Thermal ROM

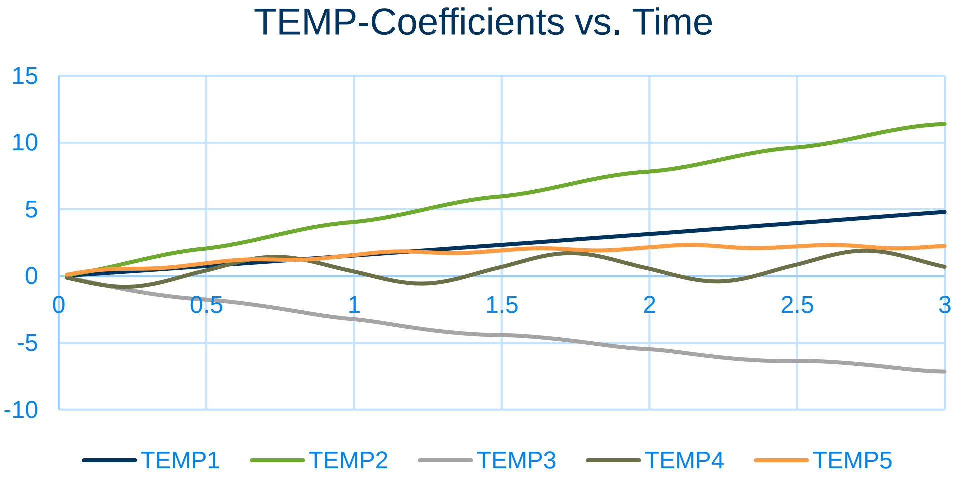
- **Static** interaction: from actual temperature coefficients, inductor position and current, the heat generation coefficients are found
- **Nonlinear**: outputs are found from response surface calculation of inputs
- **optiSLang** creates **M**etamodel of **O**ptimal **P**rognosis

- **Transient** behaviour: state space model
- **Linear**: matrices A, B, C describe equation of motion
- **MORiA** creates the ROM for TwinBuilder based on thermal system matrices

Temperature Distribution Projected onto Basis

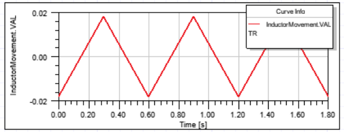




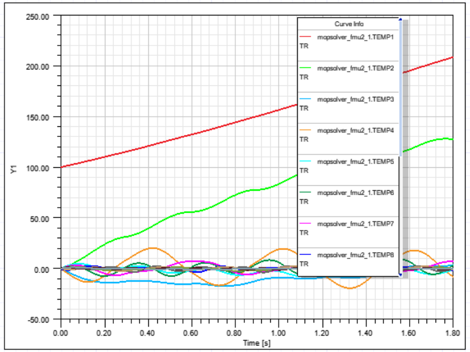


System Simulation in Twin Builder

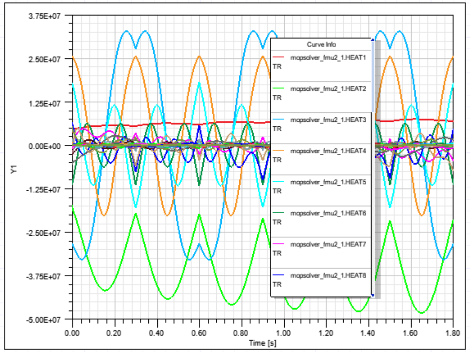
- Task:
- Compare TEMP and HEAT coefficients to those generated by coupled field simulation



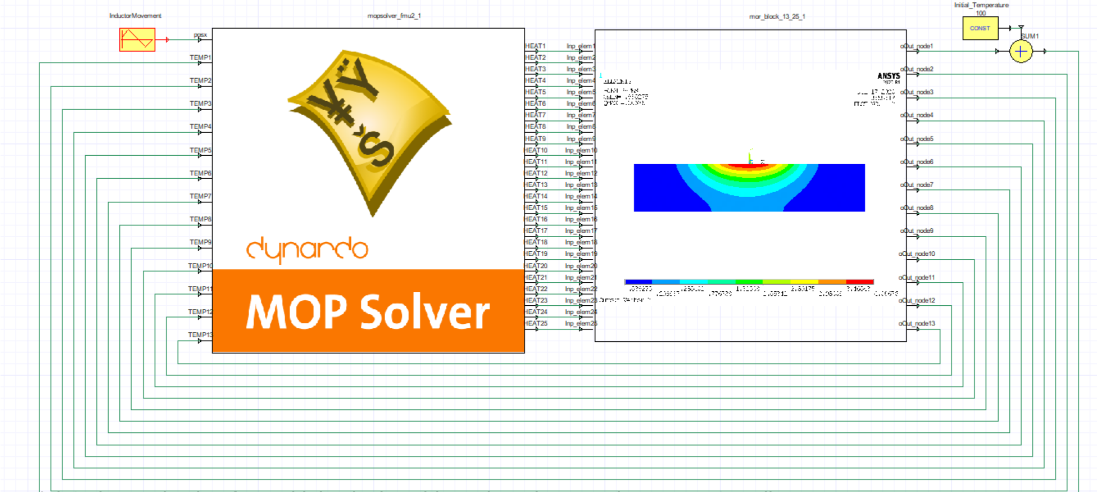
Inductor Position



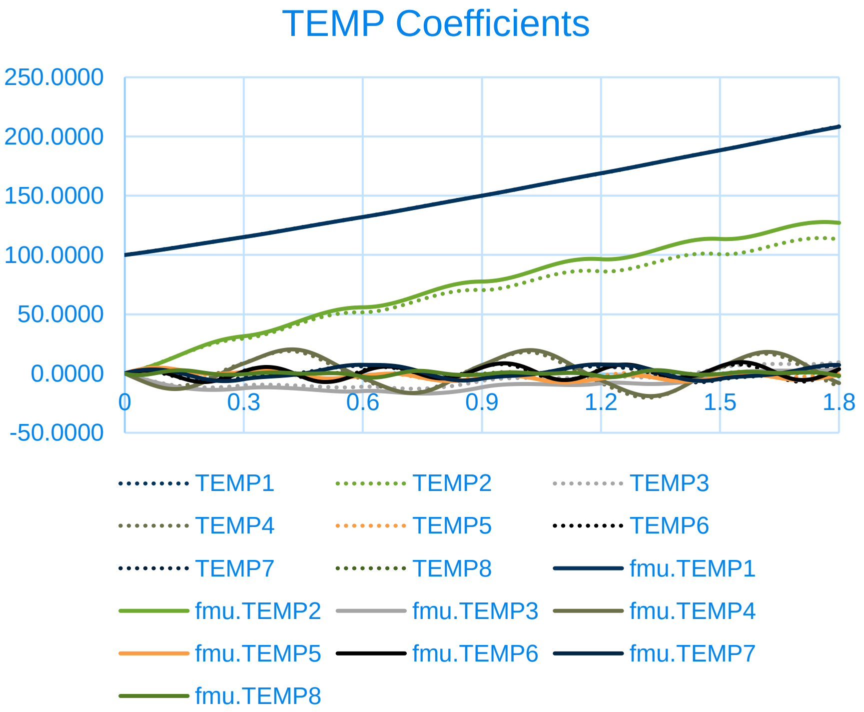
TEMP Coefficients

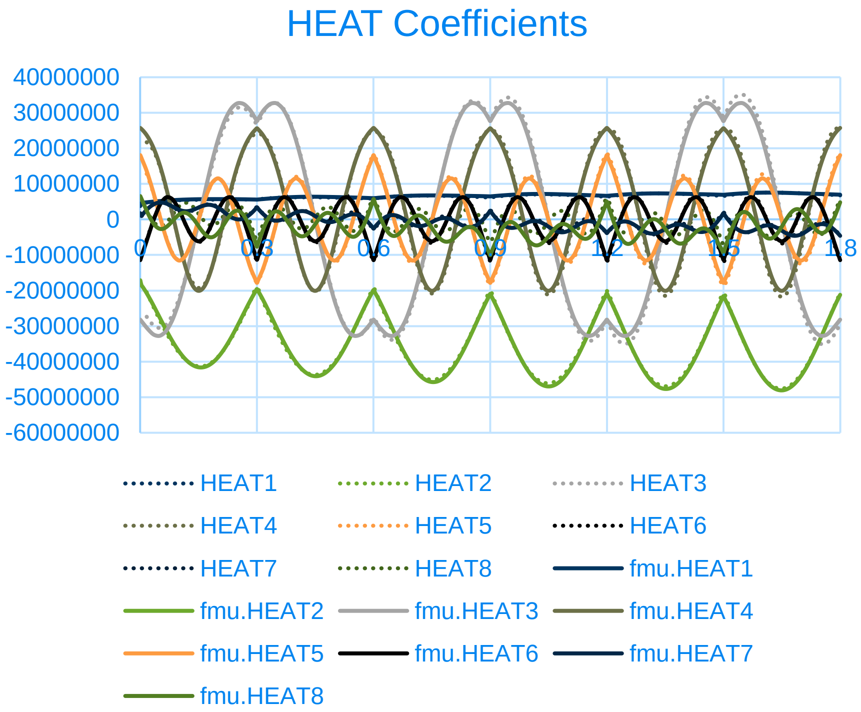


HEAT Coefficients



Reduced TEMP and HEAT Comparison





Summary

- Modes for all physical domains
- Modes for equation of motion
- Modes for coupling
- Modes as load vectors for ROM generation
- Modes as DOF in system simulation