Projection Based Model Order Reduction for Multiphysical Problems

Short Course Part 1
Electric-Thermal System Model for Power Electronics

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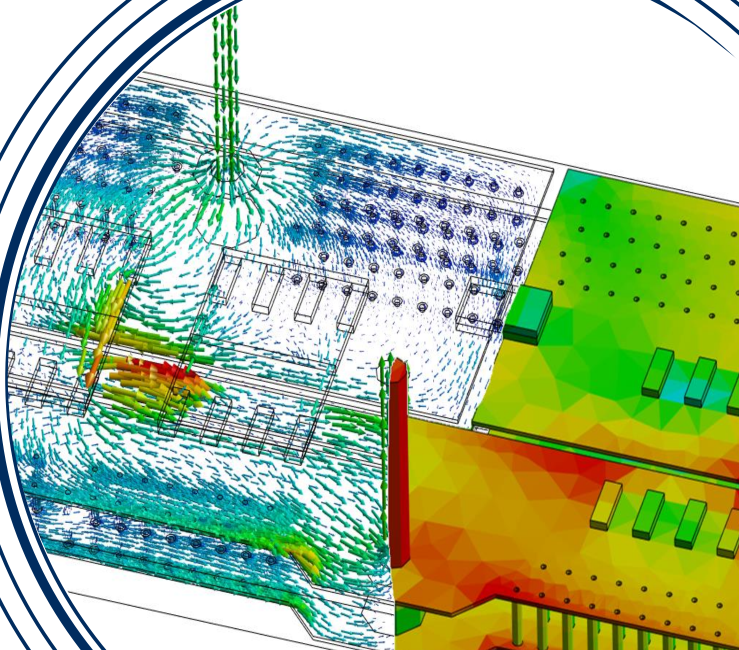

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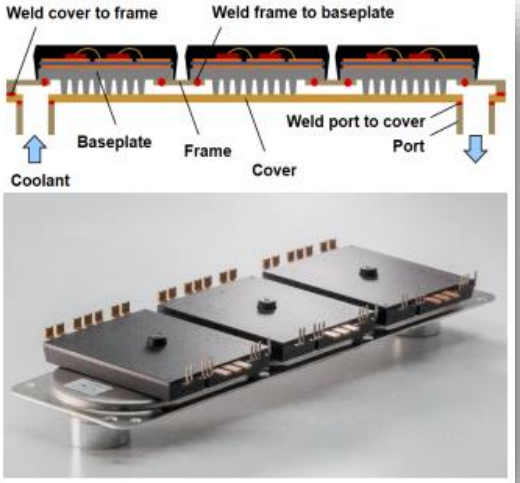
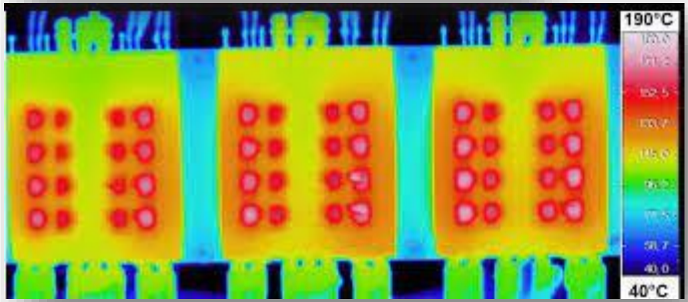
Power Electronics

- Power electronics: gets hot
- Typical applications: Inverters, bus bar systems, battery connectors
- Problem:
  - Temperature influences device characteristic and losses
  - Temperature distribution influences current distribution – new in system

- Simulation solutions:
  - Coupled field simulation electrical + thermal bidirectional
  - but: transient effects (PWM, drive cycle,…) require many time steps

  → System model required





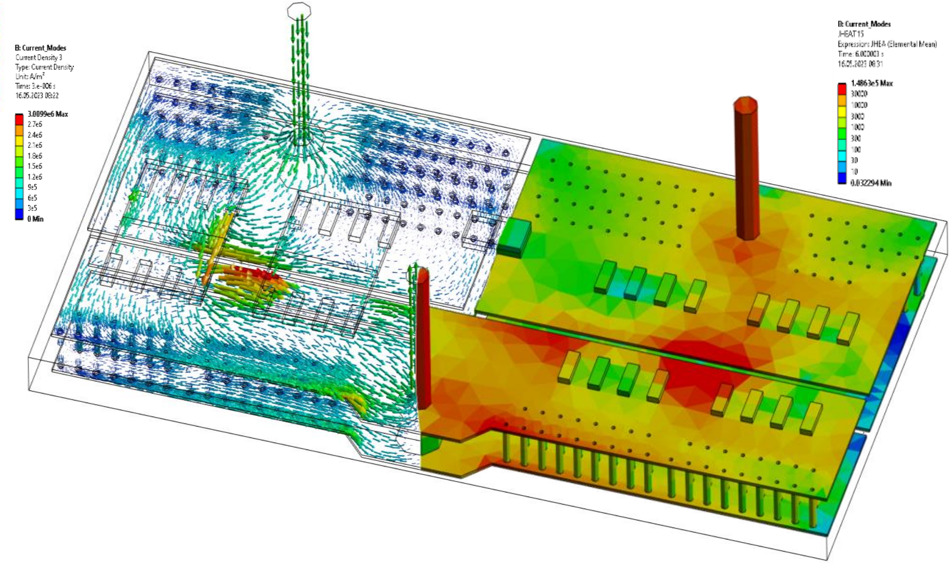
Source: High Power SiC and Si Module Platform for Automotive Traction Inverter, J Schuderer et al, 2019 PCIM Europe

Overview

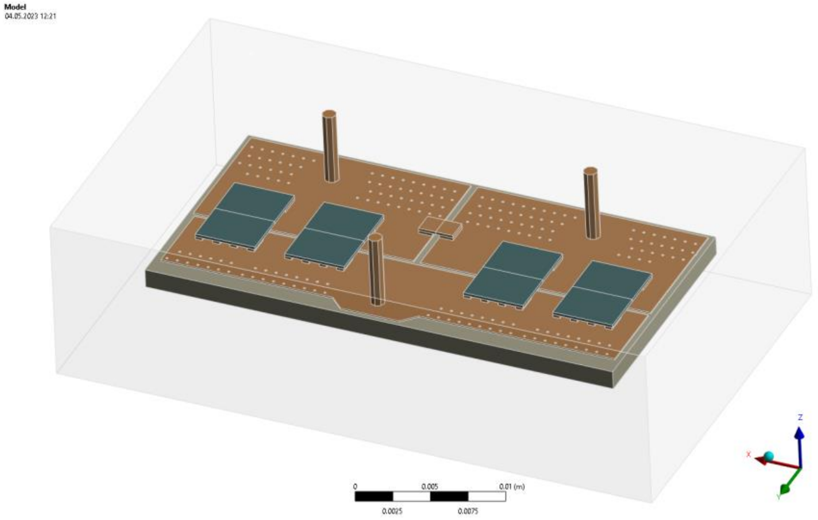
- Electric-Thermal Simulation: Field and System
  - Electric reduction: Current density modes
  - Thermal reduction: Krylov reduction
  - Coupling on system level

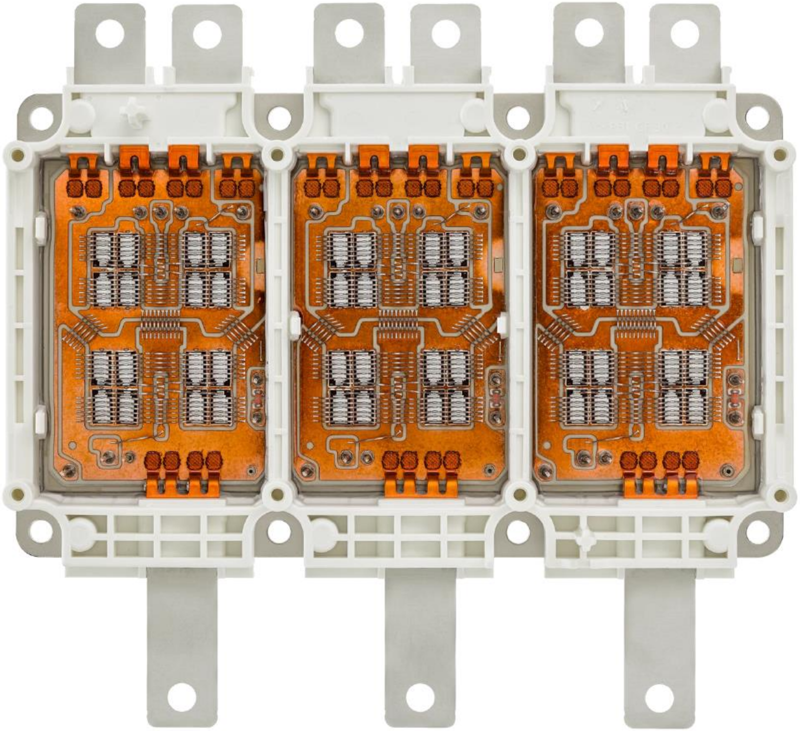
- Projection: Reduce models and couple domains
  - What are modes and where do we get them from?
  - Nonlinearities? Devide and Conquer! And then couple again…

- Summary: Electric-Thermal System Model
- Open Discussion



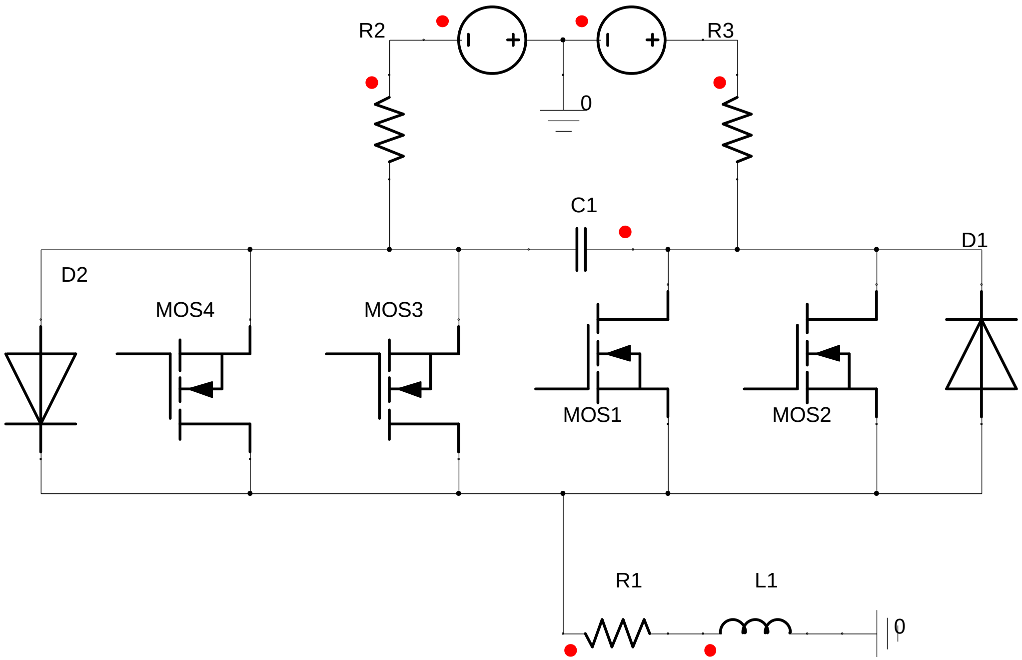
Example Half Bridge



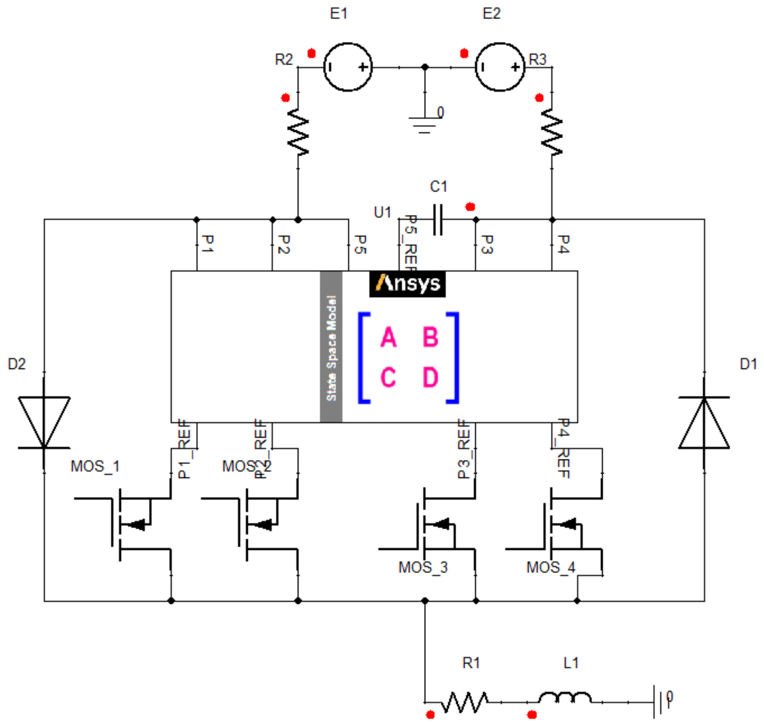


Talk Infineon at mOre driVE 2023, published in i&e, 2023

Electric Circuit Half Bridge



Embedding Parasitics Model into Circuit

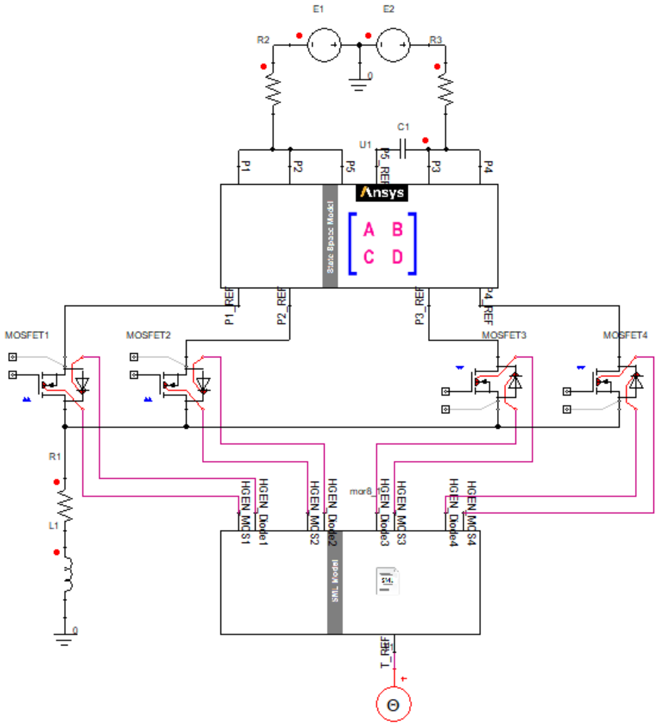


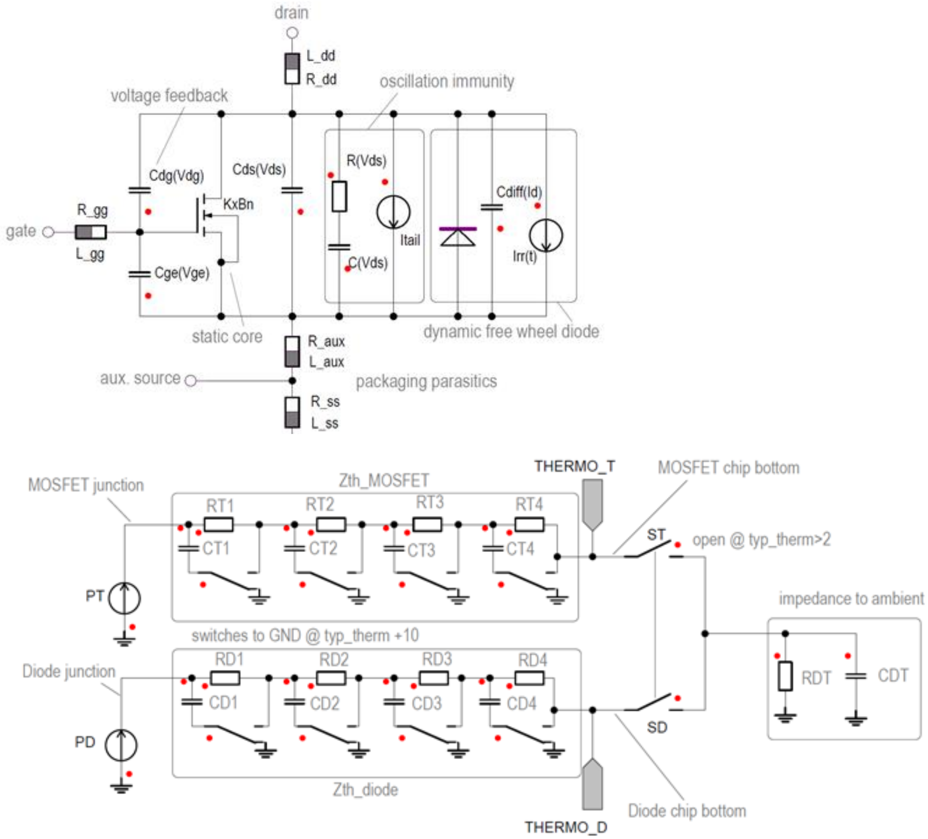
Parasitics model (RLC-Matrices)

- Conservative
- Frequency dependent
- No temperature dependence
- No heat generation

Connect MOSFET with thermal ROM

MOSFETs with thermal ports





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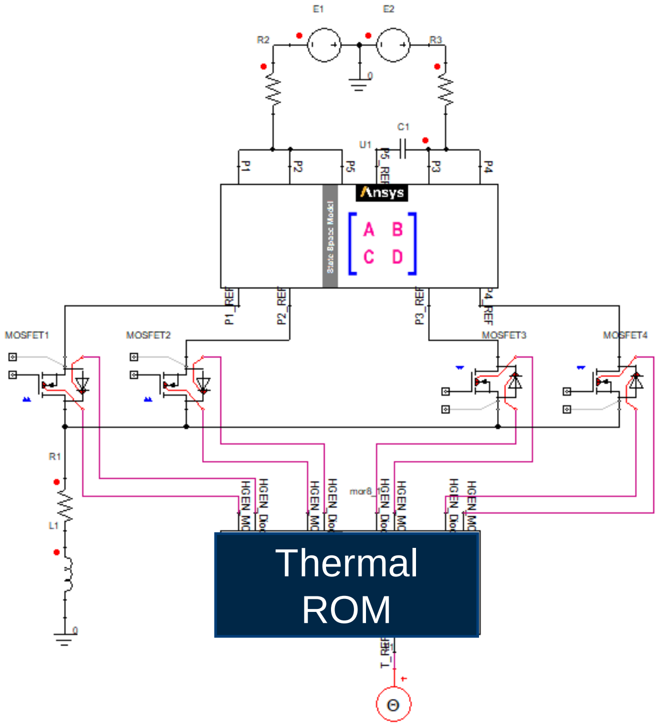
MOSFETs with thermal ports



- MOSFETs with thermal ports
- Connected to thermal ROM of PCB
- FETs as heat generation ports

Thermal ROM

Data fitting: Linear Time Invariant models (LTI)

- Time series fit from CFD or thermal field simulation (FEM)
- Input signals not necessarily a step function
- Definition of inputs (heat flow) and outputs (temperatures)
- One analysis per input

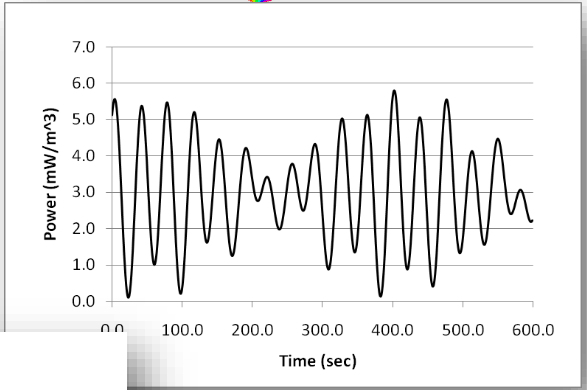
- Thermal field analysis:
  - Transient temperature field analysis with fixed boundary conditions

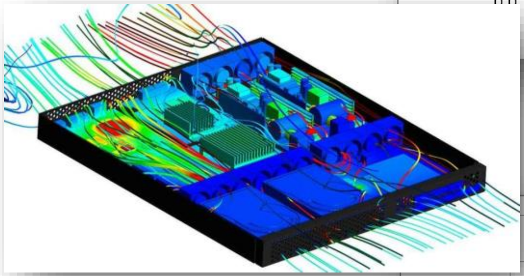
- CFD Simulation:
  - Conjugate heat transfer with constant mass flow

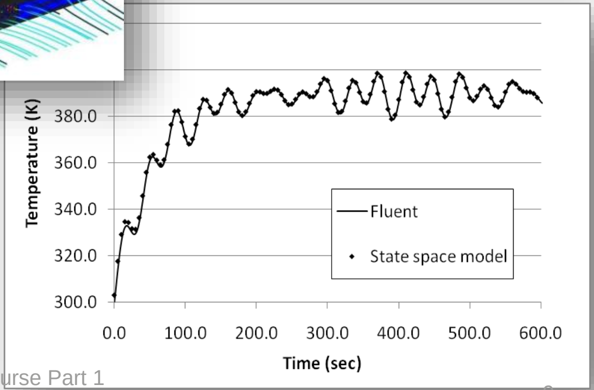
- Resulting model:
  - state space model (linear)
  - no temperature depending material properties

- Advantages:
  - Spatial resolution of HTCs due to fluid flow + advection due to mass transport is considered by default

- Disadvantages:
  - One transient simulation per input → numerically very expensive







Thermal ROM

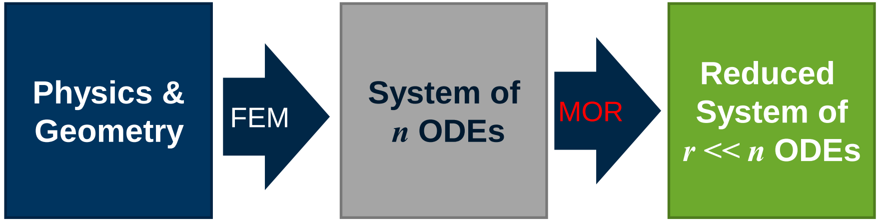
Projection onto a subspace: Model Order Reduction

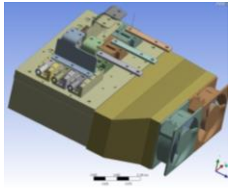
- Starting Point: Thermal field simulation (FEM)
  - Definition of boundary conditions
  - Definition of inputs (loads = heat fluxes/ heat generations/power densities, temperatures, HTCs)
  - Definition of outputs (temperatures, heat fluxes)
- Export of system matrices
- Reduction: Via Projection

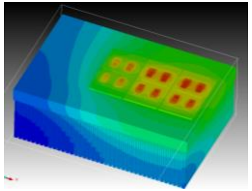
- Resulting model:
  - State Space model (linear)
  - Consider parameters:
  - Temperature depending material properties
  - Variable mass flow

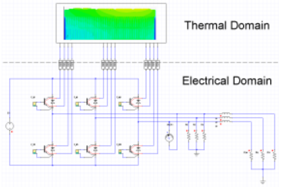
- Advantages:
  - Very fast with high quality
  - No (transient) simulation run required

- Disadvantages:
  - Complex HTC distributions need to be evaluated and mapped from CFD









Projection at a glance

transient equation of heat conduction :

$$\mathbf{C}_{\text{th}}\,\dot{\boldsymbol{\vartheta}} + \mathbf{K}_{\text{th}}\boldsymbol{\vartheta} = \mathbf{Q}$$

matrix of heat capacities — vector of temperature derivatives — matrix of heat conductivities — vector of temperatures — matrix of heat input

Reshape equation to explicit state space form:

$$\dot{\boldsymbol{\vartheta}} = -\mathbf{C}_{\text{th}}^{-1}\,\mathbf{K}_{\text{th}}\boldsymbol{\vartheta} + \mathbf{C}_{\text{th}}^{-1}\,\mathbf{Q} \qquad \dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u}$$

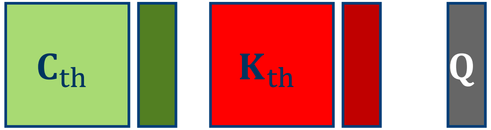
$$\boldsymbol{\vartheta} = \mathbf{I}\,\boldsymbol{\vartheta} \qquad \mathbf{y} = \mathbf{C}\mathbf{x}$$

→ $\mathbf{A} = -\mathbf{C}_{\text{th}}^{-1}\,\mathbf{K}_{\text{th}}\,, \mathbf{B} = \mathbf{C}_{\text{th}}^{-1}$ Characterization of system dynamics

→ $\mathbf{C} = \mathbf{I}$ statevector $\boldsymbol{\vartheta}$ == outputvector.

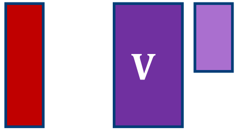
Projection at a glance

$$\mathbf{C}_{\mathrm{th}}\,\dot{\boldsymbol{\vartheta}} + \mathbf{K}_{\mathrm{th}}\,\boldsymbol{\vartheta} = \mathbf{Q}$$



Approximation of vector of nodal temperatures:

$$\boldsymbol{\vartheta} \approx \mathbf{V}\,\hat{\boldsymbol{\vartheta}}$$



Projection:

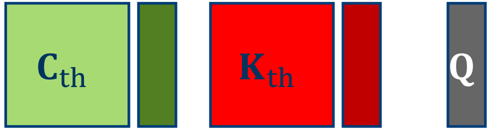
$\mathbf{V}$: Projection Matrix

$\hat{\boldsymbol{\vartheta}}$: Temperatures in Subspace

$$\begin{Bmatrix} \vartheta_1 \\ \vartheta_2 \\ \vartheta_3 \\ \vartheta_4 \\ \cdots \\ \vartheta_M \end{Bmatrix} \approx \begin{Bmatrix} v_{1\_1}\cdot\hat{\vartheta}_1 + v_{2\_1}\cdot\hat{\vartheta}_2 + v_{3\_1}\cdot\hat{\vartheta}_3 \\ v_{1\_2}\cdot\hat{\vartheta}_1 + v_{2\_2}\cdot\hat{\vartheta}_2 + v_{3\_2}\cdot\hat{\vartheta}_3 \\ v_{1\_3}\cdot\hat{\vartheta}_1 + v_{2\_3}\cdot\hat{\vartheta}_2 + v_{3\_3}\cdot\hat{\vartheta}_3 \\ v_{1\_4}\cdot\hat{\vartheta}_1 + v_{2\_4}\cdot\hat{\vartheta}_2 + v_{3\_4}\cdot\hat{\vartheta}_3 \\ \cdots \\ v_{1\_M}\cdot\hat{\vartheta}_1 + v_{2\_M}\cdot\hat{\vartheta}_2 + v_{3\_M}\cdot\hat{\vartheta}_3 \end{Bmatrix} = \begin{bmatrix} v_{1\_1} & v_{2\_1} & v_{3\_1} \\ v_{1\_2} & v_{2\_2} & v_{3\_2} \\ v_{1\_3} & v_{2\_3} & v_{3\_3} \\ v_{1\_4} & v_{2\_4} & v_{3\_4} \\ \cdots & \cdots & \cdots \\ v_{1\_M} & v_{2\_M} & v_{3\_M} \end{bmatrix} \cdot \begin{Bmatrix} \hat{\vartheta}_1 \\ \hat{\vartheta}_2 \\ \hat{\vartheta}_3 \end{Bmatrix}$$

Projection at a glance

$$\mathbf{C}_{\text{th}}\,\dot{\boldsymbol{\vartheta}} + \mathbf{K}_{\text{th}}\,\boldsymbol{\vartheta} = \mathbf{Q}$$



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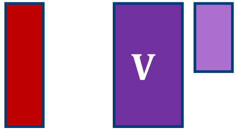
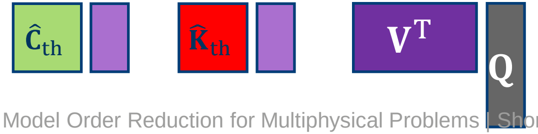


Projection:
$\mathbf{V}$: Projection Matrix
$\hat{\boldsymbol{\vartheta}}$: Temperatures in Subspace

Insert into the differential equation and multiply from the left by the transpose:

$$\mathbf{V}^{\mathrm{T}}\mathbf{C}_{\text{th}}\mathbf{V}\,\dot{\hat{\boldsymbol{\vartheta}}} + \mathbf{V}^{\mathrm{T}}\mathbf{K}_{\text{th}}\mathbf{V}\hat{\boldsymbol{\vartheta}} = \mathbf{V}^{\mathrm{T}}\mathbf{Q}$$

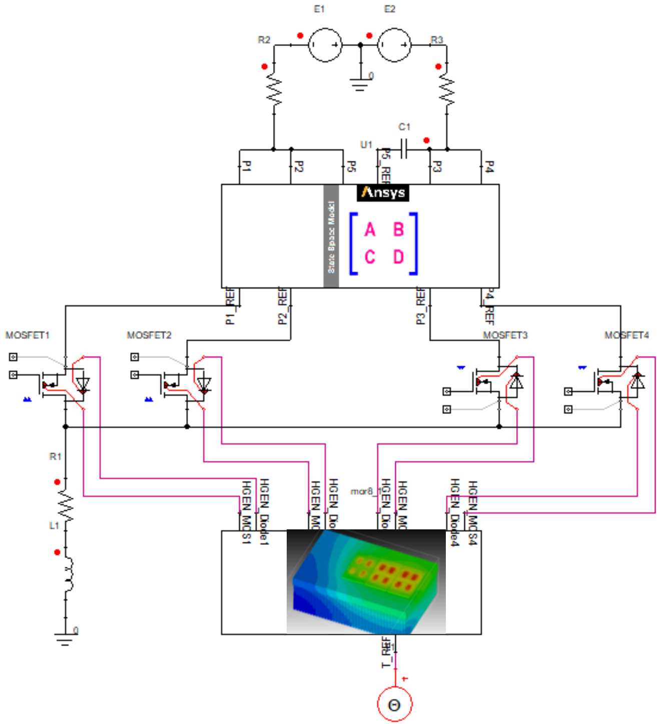
$$\hat{\mathbf{C}}_{\text{th}}\dot{\hat{\boldsymbol{\vartheta}}} + \hat{\mathbf{K}}_{\text{th}}\hat{\boldsymbol{\vartheta}} = \mathbf{V}^{\mathrm{T}}\mathbf{Q}$$



Projection at a glance

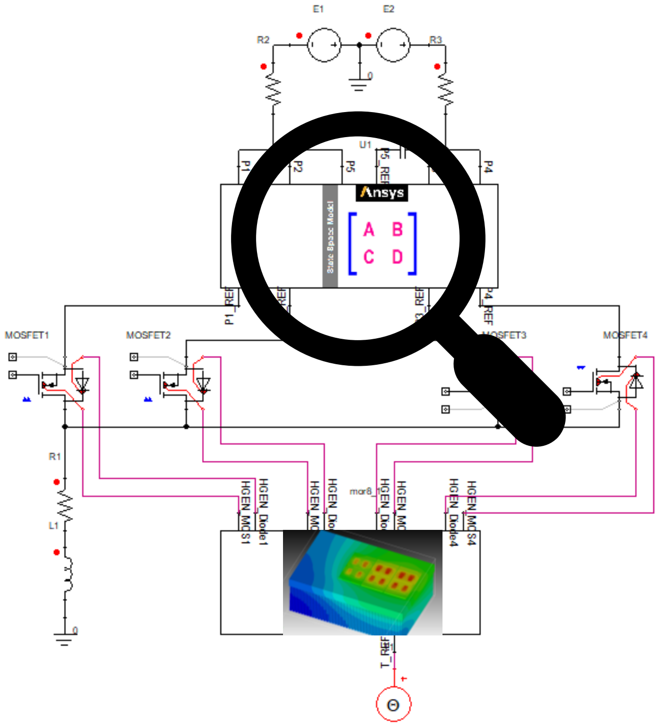
- Bring reduced model to state space format:

$$\dot{\hat{\boldsymbol{\vartheta}}} = -\hat{\mathbf{C}}_{\mathrm{th}}^{-1}\,\hat{\mathbf{K}}_{\mathrm{th}}\,\hat{\boldsymbol{\vartheta}} + \hat{\mathbf{C}}_{\mathrm{th}}^{-1}\mathbf{V}^{\mathrm{T}}\mathbf{Q}$$
$$\boldsymbol{\vartheta} = \mathbf{V}\,\hat{\boldsymbol{\vartheta}}$$



Connect MOSFET with thermal ROM

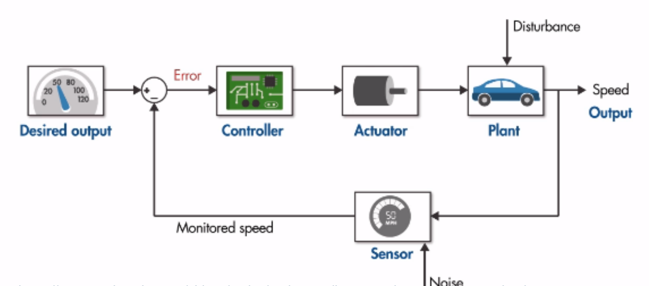
MOSFETs with thermal ports



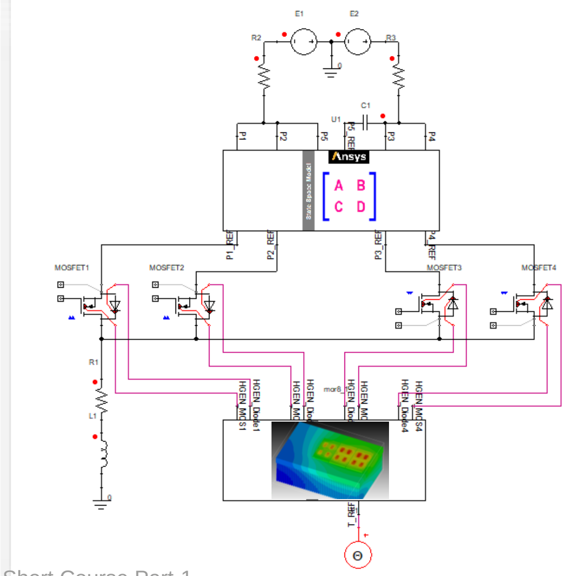
- MOSFETs with thermal ports
- Connected to thermal ROM
- FETs as heat generation ports
- Package conduction w/o thermal interaction

Field interaction on system level?

- System level simulation:
- Interaction of components via ports or terminals
- Causal modeling: Control theory → No retroactive effect on previous block → signal flow
- Conservative modeling: Like spice models → bi-directional exchange of physical quantities (current + voltage or temperature + heat) → action = reaction
- No matter of conservative or causal modeling: Exchange only of *scalar* quantities (averaged, integrated, local,…)
- electro-thermal interaction happens distributed across the entire domain → how to bring this to system level?



https://www.mathworks.com/videos/series/understanding-control-systems-123420.html



What exactly happens on field level?

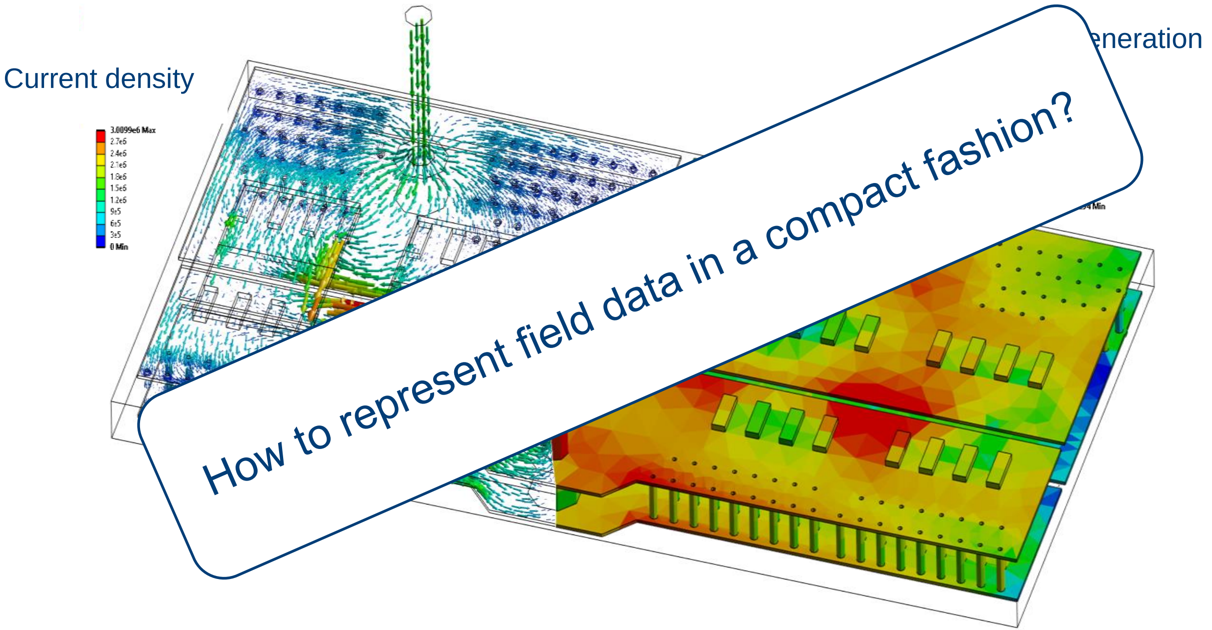
- Non uniform current distribution in the PCB
- Temperature dependent ohmic resistance
- non uniform power distribution
  → distributed heat generation
  → distributed temperature
  → distributed change in ohmic resistance
- Nonlinear distributed coupling:

$$\mathbf{q} = \rho_0(1 + \alpha\boldsymbol{\vartheta}) \cdot \mathbf{j}\circ\mathbf{j}$$

- Or in just one “location”:

$$q = \rho_0(1 + \alpha T) \cdot j^2$$

→Due to nonlinear behaviour no reduction in common step possible



Again: Projection!

$$\mathbf{u}(x,t) = \sum c_i(t) \cdot \mathbf{u}_i(x)$$

- Idea:
- Combine current density from MODES

→What are modes?

- Modes are “patterns”
- Remain constant over time → just find them once
- Span a basis covering the same subspace

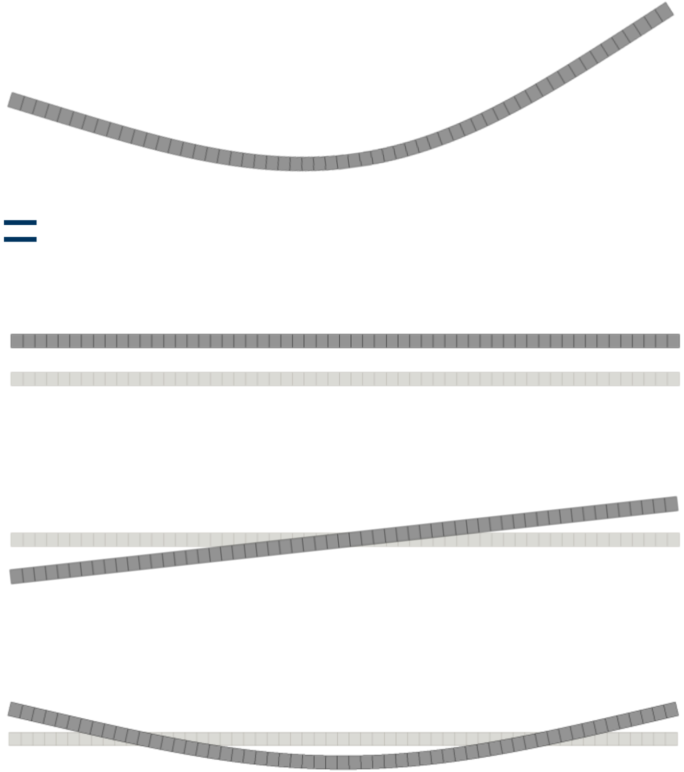
u(*x*,*t*=0.007s)

=

2.16 *

+1.65 *

+2.07 *



Projection: Determine coefficients

EuroSimE CADFEM

$$\mathbf{u}(x,t) = \sum c_i(t) \cdot \mathbf{u}_i(x)$$

Projection:

- Coefficient = Scalar product of deflection $\mathbf{u}(x,t)$ with basis vector $\mathbf{u}_i$

$$c_i(t) = \langle \mathbf{u}(x,t), \mathbf{u}_i(x) \rangle$$

→ Representation of distribution in space and time through a set of

1. Basis modes (spatial resolution, constant in time) == *n* long vectors
2. Corresponding coefficients (variing with time) == *n* (few) scalars

→ Exchange coefficients on system level via terminals to represent entire field distribution

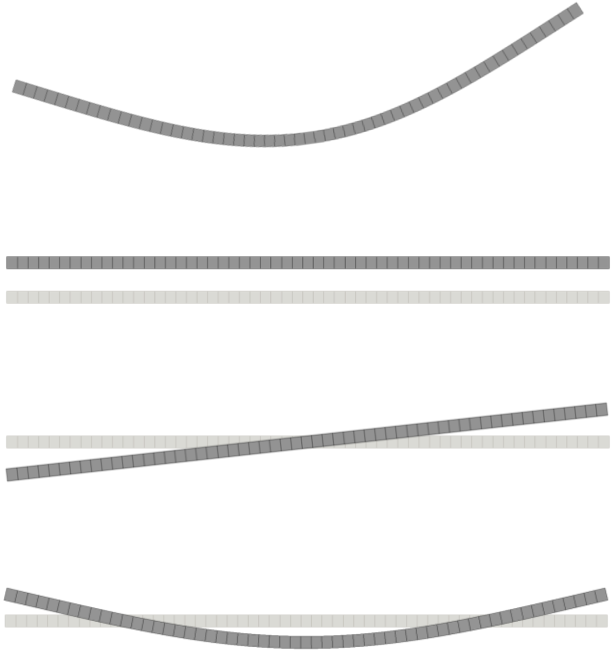
$\mathbf{u}(x,t\!=\!0.007\text{s})$

=

2.16 *

+1.65 *

+2.07 *

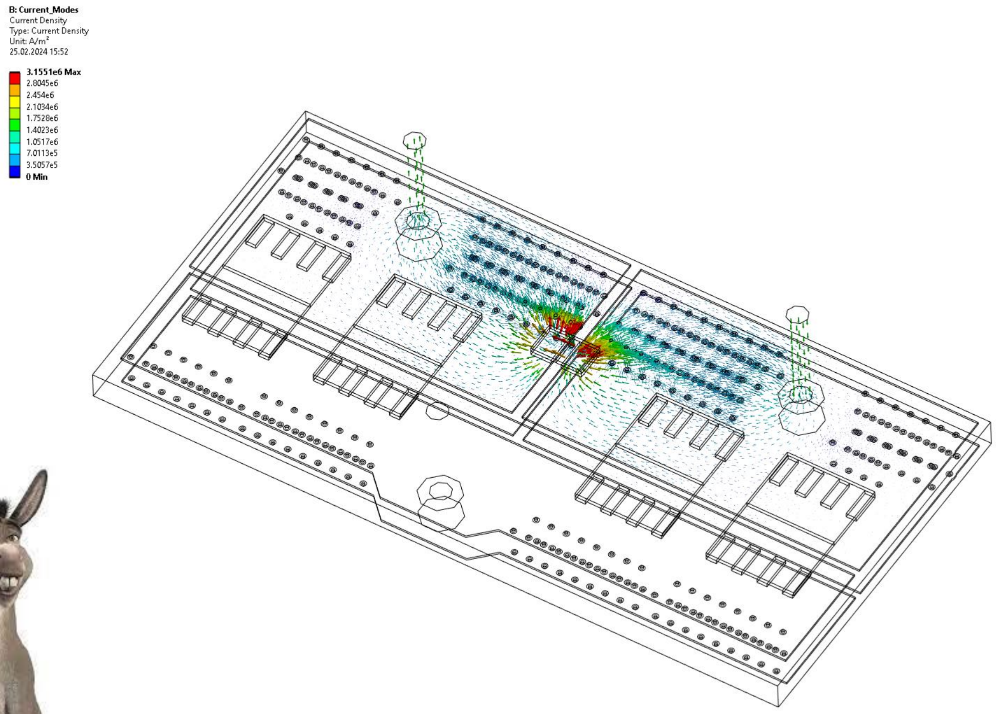


How to find modes?

- From several full field results
- Different numerical approaches:
  - POD
  - SVD
  - …
  - With further knowledge

→Here: Generate modes for current densities from different load cases

Question: Are we done?
Not quite yet!



Diagonal losses

- Goal: bring this equation to system level

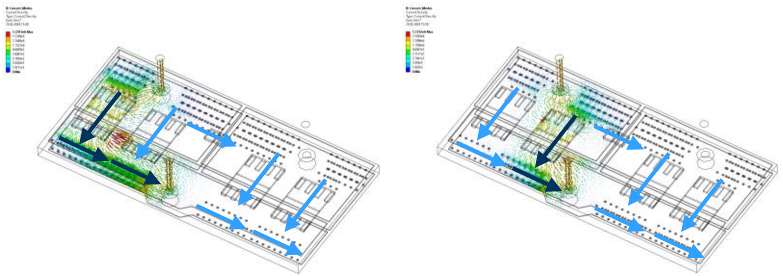
$$\mathbf{q} = \rho_0(1 + \alpha\boldsymbol{\vartheta}) \cdot \mathbf{j} \circ \mathbf{j}$$

$$Q = R_0(1 + \alpha T) \cdot I^2$$

- Couple the current density modes to heat generations
- Coupling should be diagonal

→Manipulate modes so that they are orthogonal to each other

- Orthogonality check:



- Formal check: Scalar product of current density vectors of different modes must equal zero!

$$\langle \mathbf{I}_\mathrm{i}(x,y,z), \mathbf{I}_\mathrm{j}(x,y,z) \rangle = 0$$

Diagonal losses

- Goal: bring this equation to system level

$$\mathbf{q} = \rho_0(1 + \alpha\boldsymbol{\vartheta}) \cdot \mathbf{j} \circ \mathbf{j}$$

$$Q = R_0(1 + \alpha T) \cdot I^2$$

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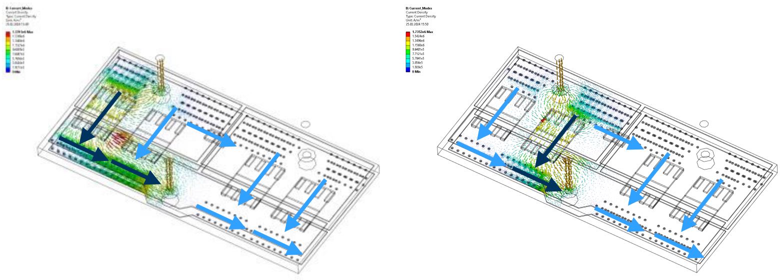
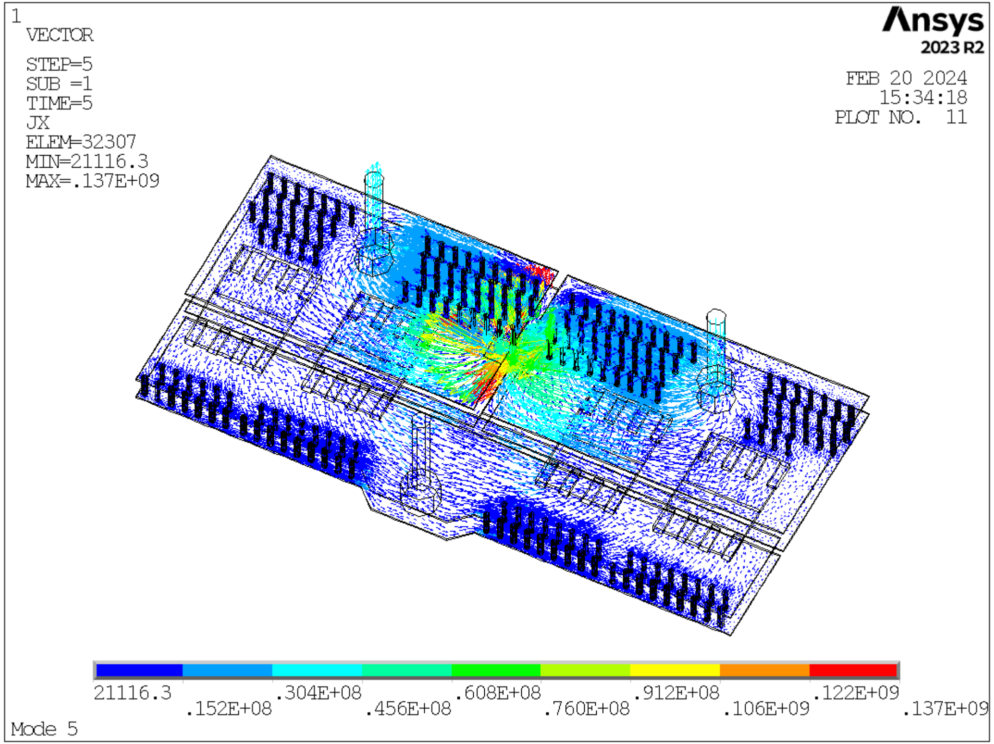


$$\mathbf{I}_1 = \begin{Bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{Bmatrix} \qquad \mathbf{I}_2 = \begin{Bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{Bmatrix}$$

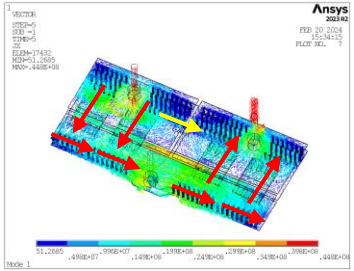
$$\langle \mathbf{I}_1, \mathbf{I}_2 \rangle = 1 \cdot 0 + 0 \cdot 1 + 0 \cdot 0 + 0 \cdot 0 + 1 \cdot 0 + 1 \cdot 1 + 0 \cdot 0 + 0 \cdot 0 + 0 \cdot 0 = 1$$

Orthogonal current density modes

- Splitting up modes in common and differential mode
- Orthogonalization procedure: Gram-Schmidt
- Normalize modes such, that $\langle \mathbf{I}_i, \mathbf{I}_i \rangle = 1$

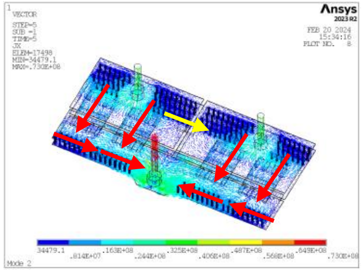


Orthogonal?



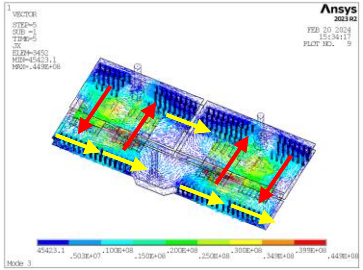
Differential mode

$$\mathbf{I}_1 = \begin{Bmatrix} + \\ + \\ - \\ - \\ + \\ + \\ + \\ + \\ 0 \end{Bmatrix}$$



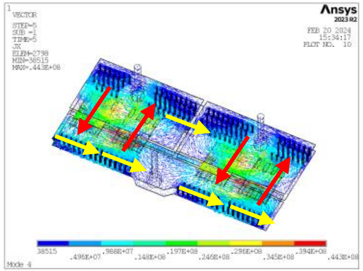
Common mode

$$\mathbf{I}_2 = \begin{Bmatrix} + \\ + \\ + \\ + \\ + \\ + \\ - \\ - \\ 0 \end{Bmatrix}$$



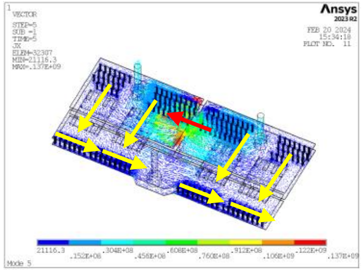
Circular sym. mode

$$\mathbf{I}_3 = \begin{Bmatrix} + \\ - \\ - \\ + \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix}$$



Circular asym. mode

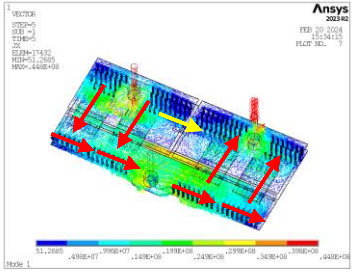
$$\mathbf{I}_4 = \begin{Bmatrix} + \\ - \\ + \\ - \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix}$$



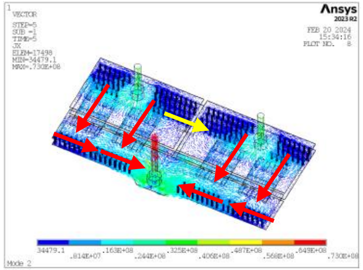
Capacitor mode

$$\mathbf{I}_5 = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ - \end{Bmatrix}$$

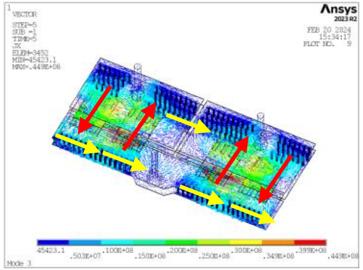
Orthogonal?



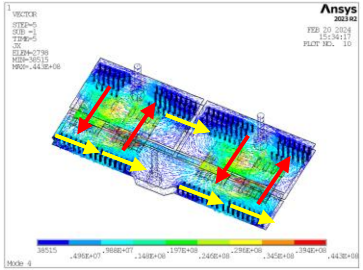
Differential mode



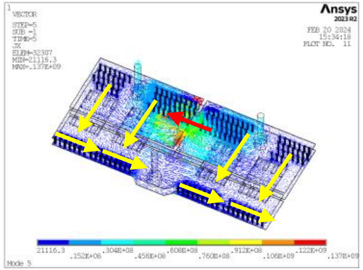
Common mode



Circular sym. mode



Circular asym. mode



Capacitor mode

$$\mathbf{I}_1 = \begin{Bmatrix} + \\ + \\ - \\ - \\ + \\ + \\ + \\ + \\ 0 \end{Bmatrix}$$

$$\mathbf{I}_2 = \begin{Bmatrix} + \\ + \\ + \\ + \\ + \\ + \\ - \\ - \\ 0 \end{Bmatrix}$$

$\begin{pmatrix} + \\ - \end{pmatrix}$ $\begin{pmatrix} + \\ - \end{pmatrix}$ $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$

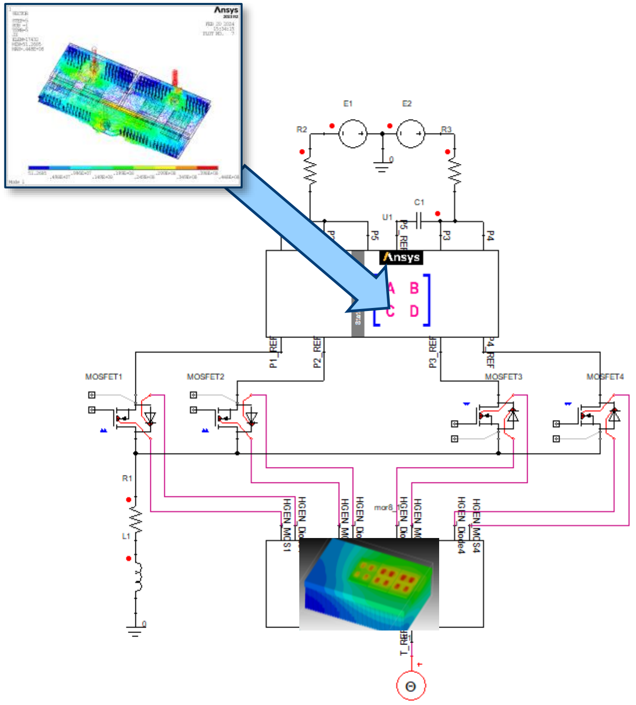
$\langle \mathbf{I}_i, \mathbf{I}_j \rangle$	$\mathbf{I}_1$	$\mathbf{I}_2$	$\mathbf{I}_3$	$\mathbf{I}_4$	$\mathbf{I}_5$
$\mathbf{I}_1$	1	0	0	0	0
$\mathbf{I}_2$		1	0	0	0
$\mathbf{I}_3$			1	0	0
$\mathbf{I}_4$				1	0
$\mathbf{I}_5$					1

Are we there yet?

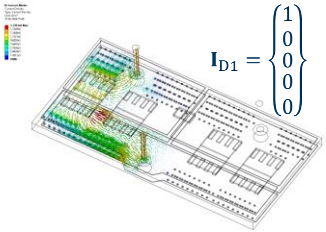
What we got and what is missing

- Simulation for electric behaviour of devices
- Orthonormal current density modes inside the PCB

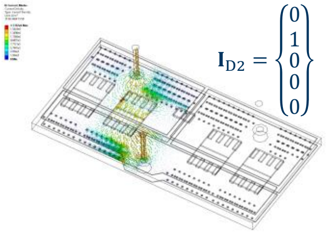
- ROM for thermal behaviour – but only with scalar terminals
- No thermo-electrical interaction on field level yet
- No correlation between device currents and current density modes



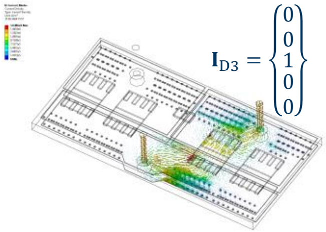
Transformation matrix linking device currents to current density modes



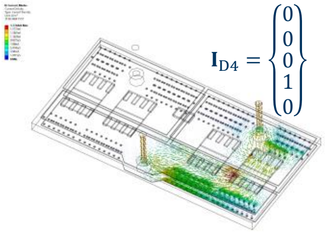
$$\mathbf{I}_{D1} = \begin{Bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix}$$



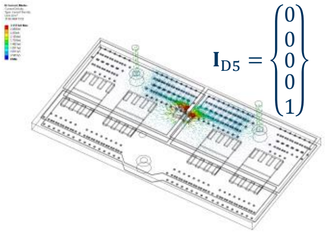
$$\mathbf{I}_{D2} = \begin{Bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{Bmatrix}$$



$$\mathbf{I}_{D3} = \begin{Bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{Bmatrix}$$



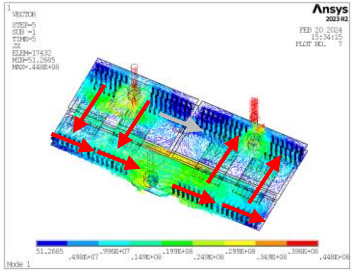
$$\mathbf{I}_{D4} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{Bmatrix}$$

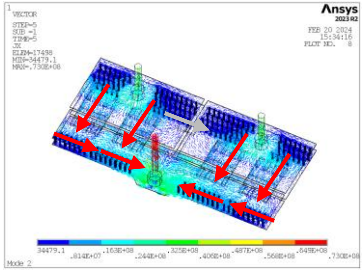


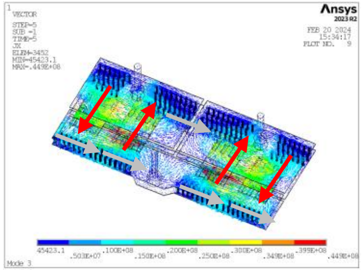
$$\mathbf{I}_{D5} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{Bmatrix}$$

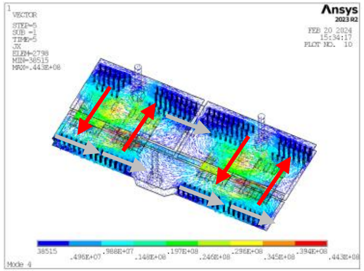
- Split up modes
- Orthogonalize them

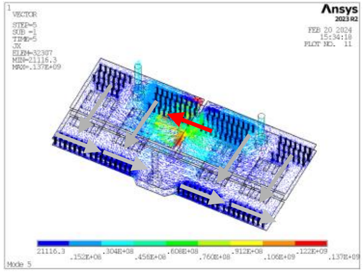
- Construct vectors representing device currents
- Subject them to same steps as current distribution vectors
- Add them up to a transformation matrix **T**



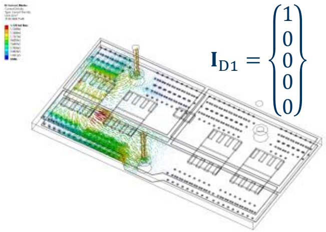




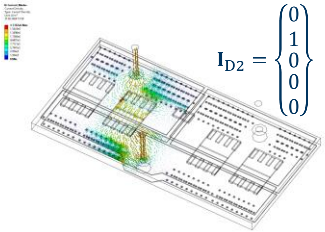




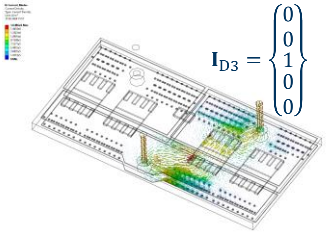
Transformation matrix linking device currents to current density modes



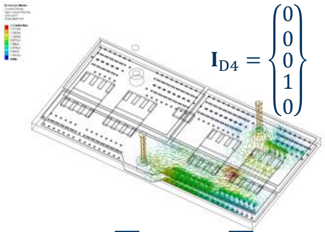
$\mathbf{I}_{D1} = \begin{Bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix}$



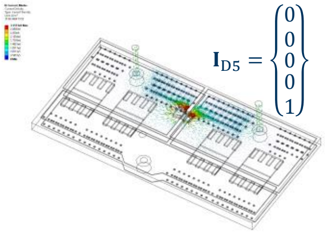
$\mathbf{I}_{D2} = \begin{Bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{Bmatrix}$



$\mathbf{I}_{D3} = \begin{Bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{Bmatrix}$



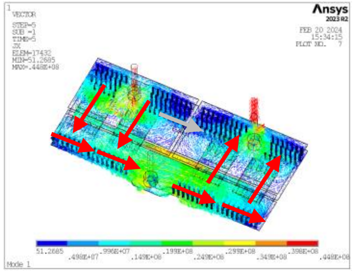
$\mathbf{I}_{D4} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{Bmatrix}$

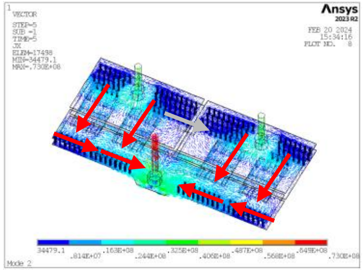


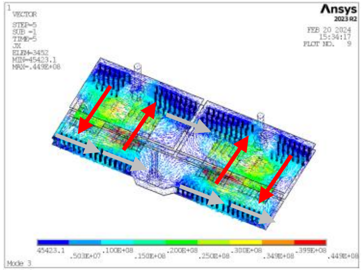
$\mathbf{I}_{D5} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{Bmatrix}$

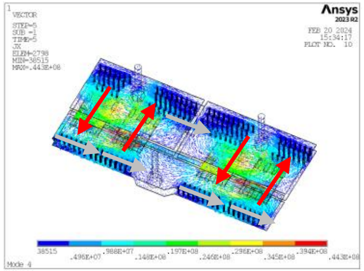
- Split up modes
- Orthogonalize them

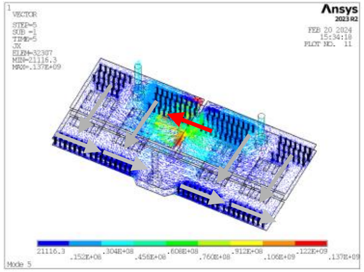
$$\begin{Bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \end{Bmatrix} = \begin{bmatrix} T_{11} & T_{21} & T_{31} & T_{41} & T_{51} \\ T_{12} & T_{22} & T_{32} & T_{42} & T_{52} \\ T_{13} & T_{23} & T_{33} & T_{43} & T_{53} \\ T_{14} & T_{24} & T_{34} & T_{44} & T_{54} \\ T_{15} & T_{25} & T_{35} & T_{45} & T_{55} \end{bmatrix} \cdot \begin{Bmatrix} I_{m1} \\ I_{m2} \\ I_{m3} \\ I_{m4} \\ I_{m5} \end{Bmatrix}$$





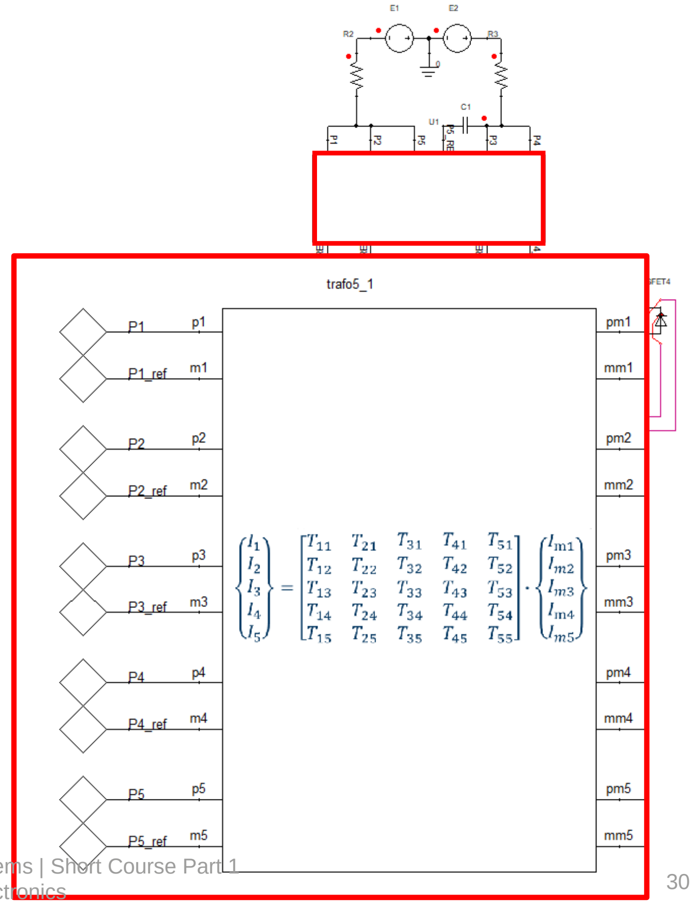






What we got and what is missing

- Simulation for electric behaviour of devices
- Orthonormal current distribution modes inside the PCB
- Transformation matrix linking device currents and current distribution modes



- ROM for thermal behaviour – but only with scalar terminals
- No thermo-electrical interaction on field level yet

Thermo-electrical field interaction on system level

- Diagonal losses for each mode:

$$U = R_0(1 + \alpha T) \cdot I$$
$$Q = U \cdot I = R_0(1 + \alpha T) \cdot I^2$$

- Implementation as temperature dependent ohmic resistance for each current mode
- Heat generation is defined as power loss
- Conservative behaviour couples thermal to electrical domain

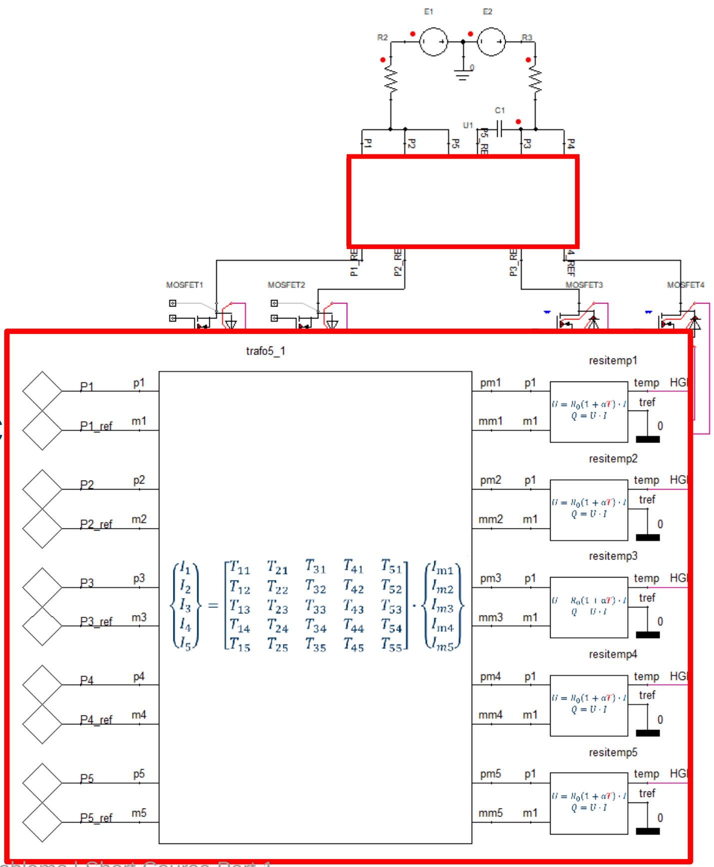
- Modes were normalized such, that $\mathrm{R}_0 = 1$

```
LIBRARY IEEE;
use Ieee.thermal_systems.ALL;
USE IEEE.ELECTRICAL_SYSTEMS.ALL;
ENTITY ResiTemp IS
  GENERIC (
    alpha : REAL := 0.004);
  PORT (
    TERMINAL p1,m1 : ELECTRICAL;
    TERMINAL Temp,TRef : THERMAL);
END ENTITY ResiTemp;
```

```
ARCHITECTURE behav OF ResiTemp IS
  QUANTITY v ACROSS i THROUGH p1 TO m1;
  QUANTITY T ACROSS Heat THROUGH Temp TO TRef;
BEGIN
  v==(1+alpha*(T))*i;
  heat==-i*v;
END ARCHITECTURE behav;
```

What we got and what is missing

- Simulation for electric behaviour of devices
- Orthonormal current distribution modes inside the PCB
- Transformation matrix linking device currents and current distribution modes
- thermo-electrical interaction on field level realized through conservative modal coupling: Modal ohmic resistor with Joule heating
- ROM for thermal behaviour – but only with scalar terminals



Thermal ROM with modal inputs

Two types of losses or heat inputs to the thermal ROM

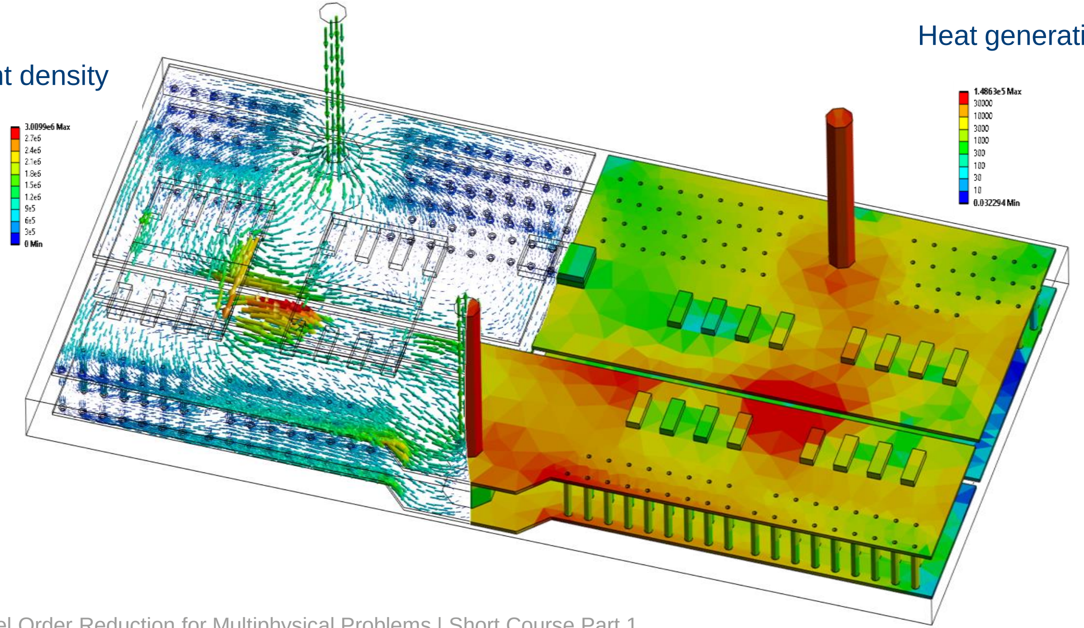
- Concentrated and mostly homogeneous losses (Device losses) → already considered
- Distributed and inhomogeneous losses (Joule heating):

$$Q = R_0(1 + \alpha T) \cdot I^2 = k \cdot I^2$$

- Scaling of squared current density modes

Current density

Heat generation



Thermal ROM with modal inputs

- ROM should behave in a conservative fashion:
- For each load vector, a corresponding output vector exists

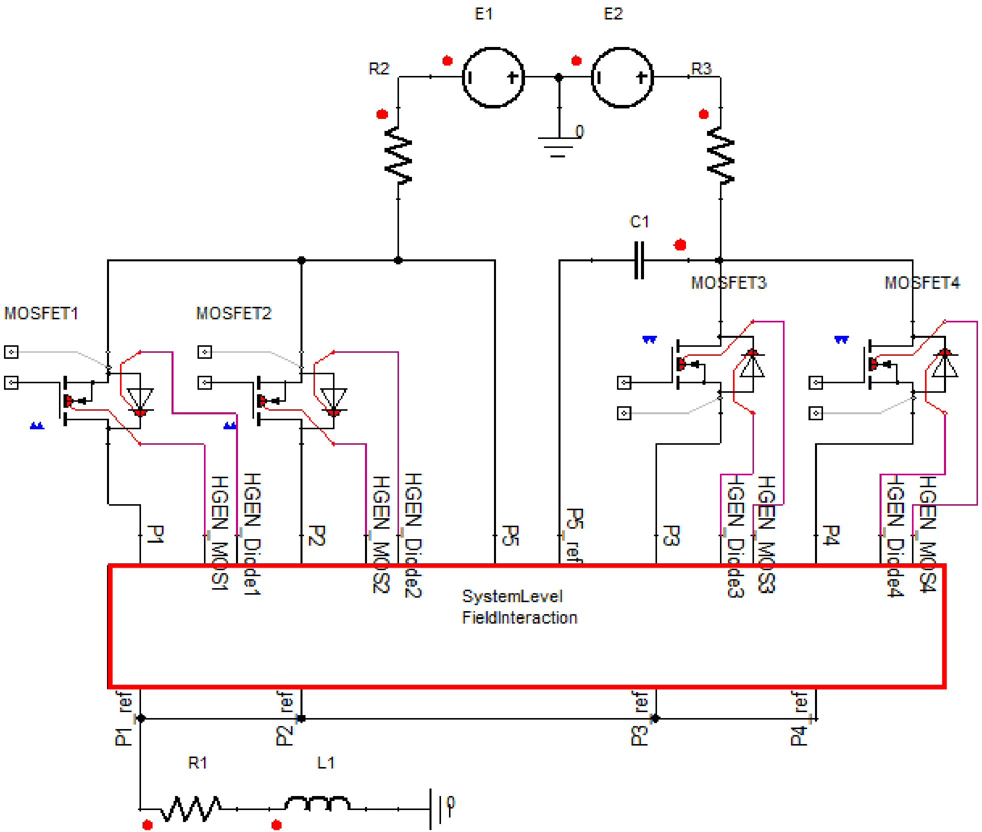
- Heat distribution delivers corresponding temperature distribution

- By adding modal heat load vectors, we can observe the corresponding temperature distribution in the same basis

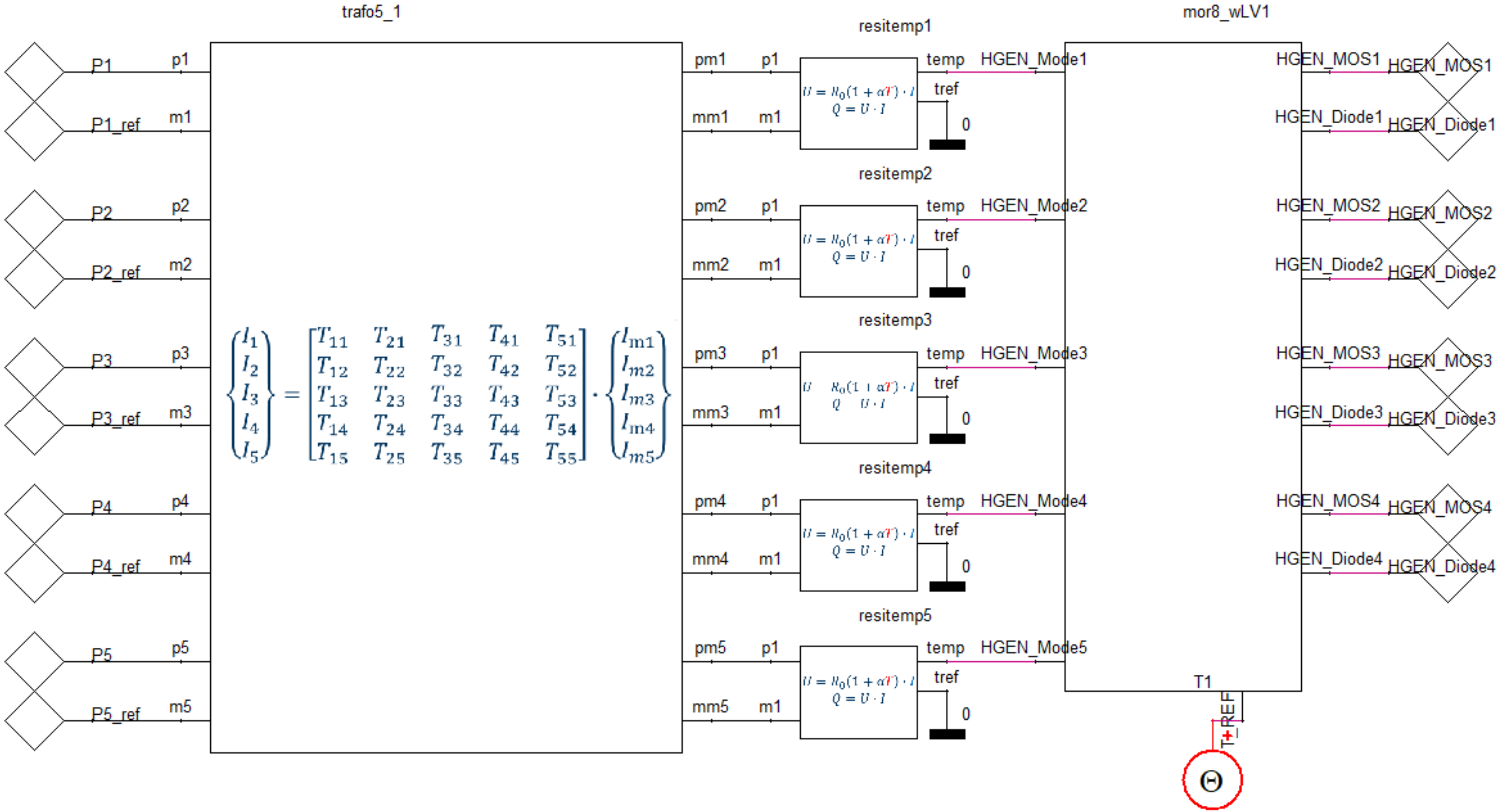
- As heat load vectors, we use scaled squared current density modes

$$Q = R_0(1 + \alpha T) \cdot I^2 = k \cdot I^2$$

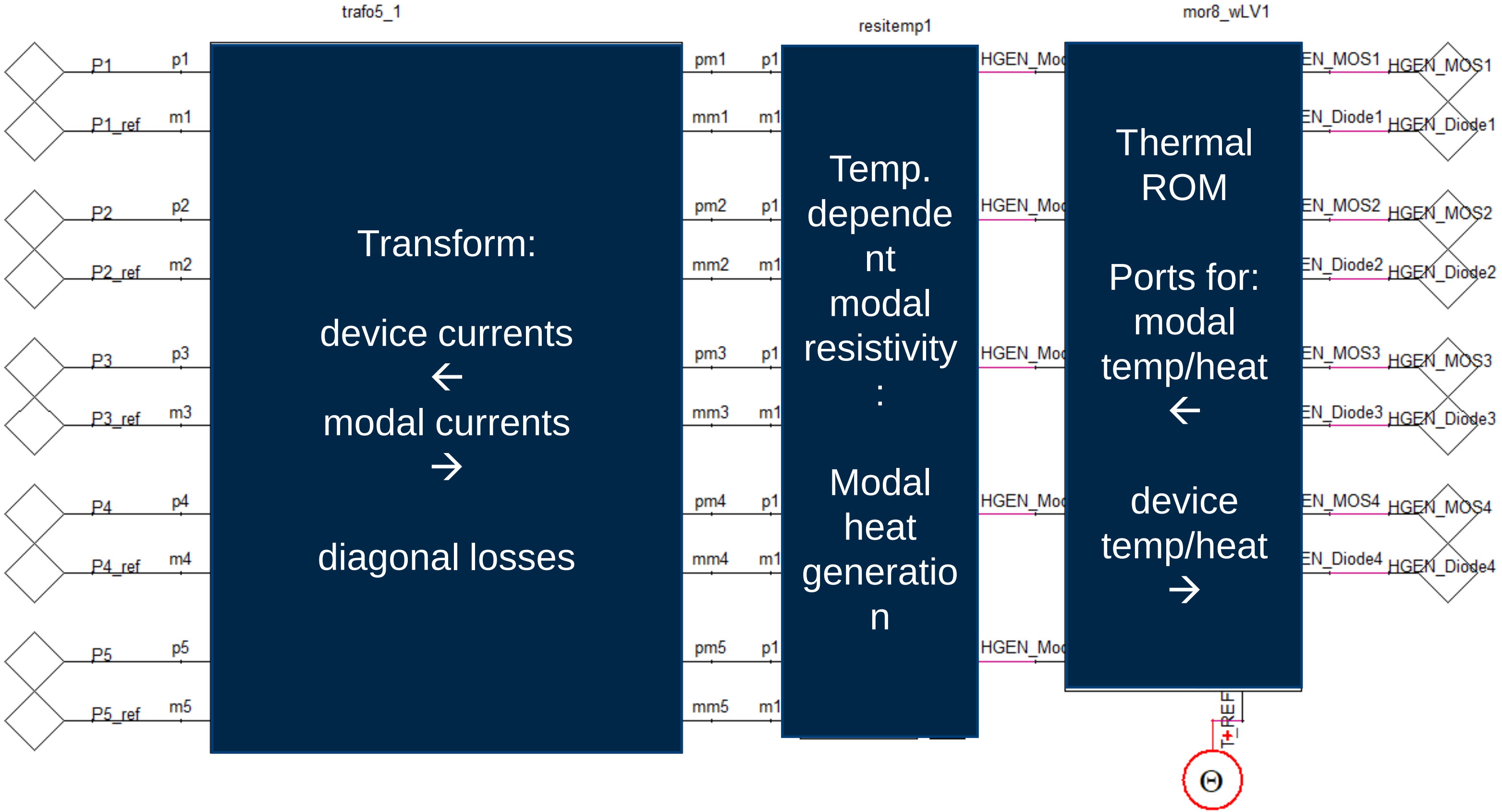
Coupling on system level



Coupling on system level



Coupling on system level

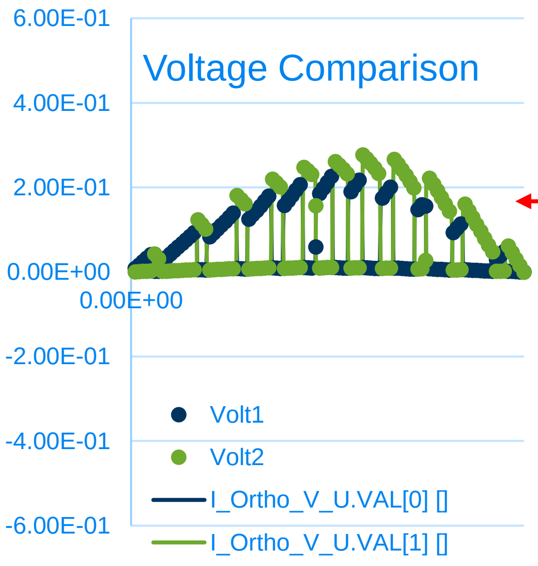


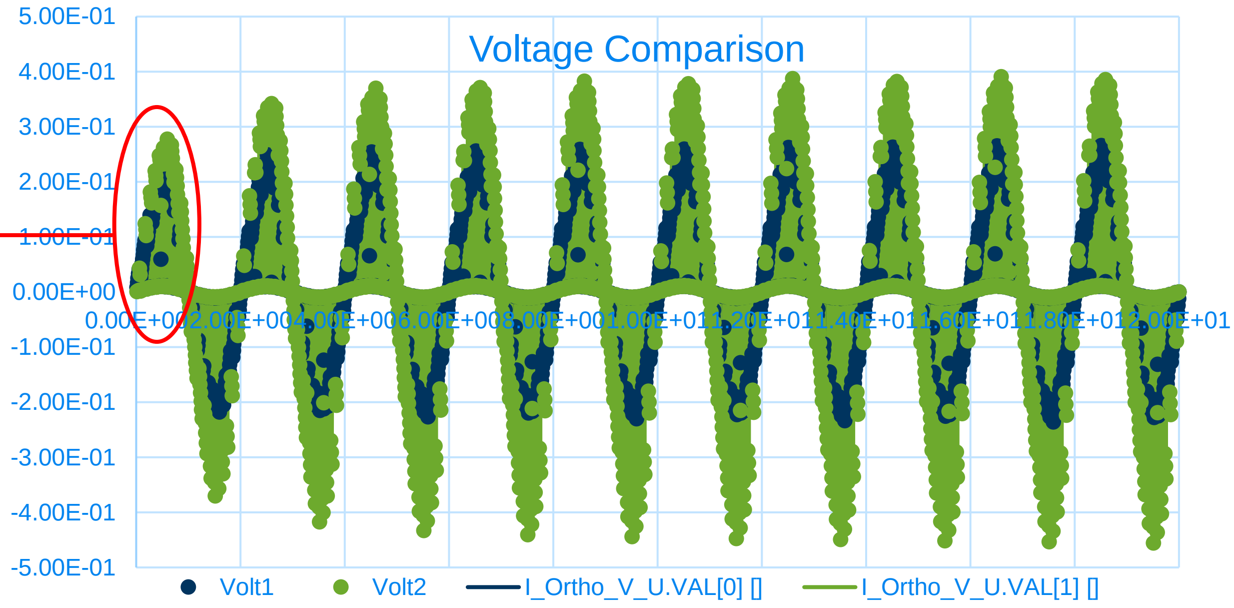
Verification w/o Skin/Proximity Effects

Coupled field solution, dt = 10 ms: Elapsed Time (sec) = 4515.0

Coupled System Solution, dt = 1 ms: Solution Process 00:00:01.575

30,000 * faster





Training Effort

From which Simulation emerges which Reduced Order Model?

Electrical Simulation

- Steady state simulation
- One solution per load case
- Results: Current density distributions

Post processing:

- Split into modes and orthogonalize & normalize
- Save transformation matrix: Connect device currents to current density modes

Numerical effort:

- low

Thermal Simulation

- Steady state thermal simulation setup with boundary conditions and loads
- Export system matrices
- Add load vectors for modal heat generation
- No Solution!

Post Processing:

- Automatic generation of state space model

Numerical effort:

- low

Ohmic resistance

- Manual programming (once) of temperature depending resistance

Post Processing:

- None – due to normalization of the current density modes, the modal Resistance == 1

Numerical Effort:

- None

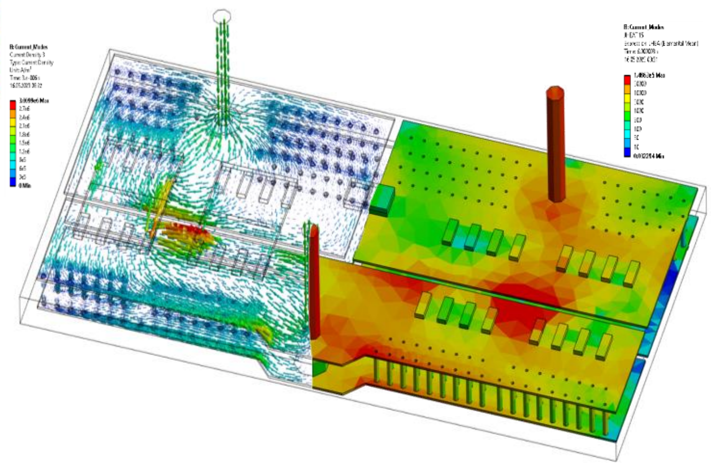
Advantages of system level simulation

- Very fast simulation compared to full field coupled simulations
- High modularity:
  - Different model types
  - Different abstraction layers
  - Easy exchange of components
  - Setup of toolboxes
- Mutual interactions of different components

→ High optimization potential

Take Aways

- System simulation for EM-thermal coupling is possible
- System simulation is fast (≈ 10000 times faster than coupled field), thus transient simulations become possible → further step towards active cycling
- Projection based model order reduction allows numerically very efficient generation of reduced order models for linear problems
- (Many) Nonlinearities can be pushed to system level



Use the method for further applications?

- YES! Definitely
- Interested? → Stay for the second part of this short course:
  - Part II/1: Numerics: Vector spaces and subspace generation
  - Part II/2: More Applications and Theory behind conservativity of models: Modes, load vectors and couplings

- Still want to learn more?
  - Monday, Session 4, 15:40: A new method of model order reduction (MOR) for mechanical non-linear elastic-plastic problems in power electronics packaging
  - Tuesday, Session 16, 17:10: Reduced-Order Model for Solder Balls – Potential of projection-based approaches for representing viscoplastic behavior